Saturation Throughput Analysis for a Wireless Network with Multiple Channels

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Glossary

ST - Station
CH - Channel
AP - Access Point
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Introduction

In our modern world transporting information between two spatially separated places without human interaction has become common use. There is a huge number of communication systems which try to achieve quick, reliable and robust information transfer. Each system is designed for one specific physical medium which is responsible for the energy transfer of the signal. There are two types of physical media namely the cabled and the wireless media. The cabled setup is usually faster and more robust than wireless communication. This is due to the good electromagnetic shielding of the cable which therefore has low probability of interaction with interference signals. In contrast the wireless setup is a broadcast medium which is shared among all users of a system. There is a greater signal to noise and interference ratio. Additionally the wireless signal can be reflected, absorbed and reach the destination on different paths. Therefore establishing a fast and reliable communication is more challenging.

This thesis introduces different wireless setups and channel models and analyses the saturation throughput theoretically and by simulation. The generality of the topic and simplicity of the assumed models suggests a large amount of related work. In the following the most influencing work is presented. In [1] a similar Markov model of the stations has been used. Additionally the saturation throughput analysis was conducted in a similar way. [1] can be interpreted as a special case of the models used in this thesis. The basement of the saturation throughput analysis is the decoupling approximation which get first introduced in [2]. The bandwidth-SINR tradeoff was motivated by the work in [3] which assumes a distribution of the locations of the stations in the 2D plane in contrast to fixed but arbitrary positions used in this thesis. Starting from similar
assumptions as in [3] an analogue bandwidth-SINR tradeoff was generated for a special setup of stations and access points. Finally [4] and [5] provide very good and rigorous fix point analysis for a similar Markov chain used in this thesis.

The Thesis is structured as follows. In Chapter 2 the model of the sender in Section 2.2 and of the receiver in Section 2.3 as well as four models of the MAC in Section 2.4 and two types of interference models in Section 2.5 are introduced. Chapter 3 deals with the analysis of the saturation throughput of combinations of the MAC and interference models presented in Chapter 2. A brief description of the simulation is given in Chapter 4. The results from analysis and simulation are compared in Chapter 5. Finally Chapter 6 gives a summary and discussion of the previous chapters.
System Model

In this section the basic model of the system and of the sender and receiver is described and the considered MAC and interference models presented. Table 2.1 lists the basic system parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>total number of stations</td>
</tr>
<tr>
<td>$c$</td>
<td>total number of channels</td>
</tr>
<tr>
<td>$m$</td>
<td>packet duration in time slots</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>duration of a time slot in seconds</td>
</tr>
<tr>
<td>$B$</td>
<td>total available bandwidth in Hertz</td>
</tr>
<tr>
<td>$R_{\text{max}}$</td>
<td>maximum transmission rate</td>
</tr>
<tr>
<td>$r$</td>
<td>total number of states of a station</td>
</tr>
<tr>
<td>$L_i$</td>
<td>state of a station with $i = 1, 2, \ldots, r$</td>
</tr>
<tr>
<td>$W_i$</td>
<td>total backoff size of state $L_i$</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>transmission power of station $\text{ST}_i$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>attenuation exponent</td>
</tr>
<tr>
<td>$d_{ij}$</td>
<td>distance between station $\text{ST}_i$ and Access Point $\text{AP}_j$</td>
</tr>
<tr>
<td>$N$</td>
<td>thermal noise power</td>
</tr>
<tr>
<td>$\alpha_{ij}$</td>
<td>cross-channel interference between channels $\text{CH}_i$ and $\text{CH}_j$</td>
</tr>
</tbody>
</table>

Table 2.1: System parameters
2. SYSTEM MODEL

2.1 Basic Setup

The basic setup consists of \( n \) stations (ST) acting as senders and \( c \) receiving access points (AP). Each access point listens on a channel with bandwidth \( B/c \) where \( B \) is the total available bandwidth. Both the senders and receivers are located arbitrarily in the 2D plane. Each station transmits with a specific power and therefore for each ST-AP pair there is a different received signal power which depends on the distance of the station from the access point and of the transmission power of the station. For the sake of simplicity of analysis and simulation time is slotted with granularity \( \sigma \) where \( \sigma \) is the duration of a time slot in seconds. In the following sections the model of the sender and receiver is described in greater detail. Furthermore the MAC models and interference models are described.

2.2 Model of Sender

We assume a backoff MAC protocol. Therefore the state of the sender is modeled as a Markov chain which is depicted in Fig. 2.1. A station can be in \( r \) possible states \( L_i \) with \( i = 1, 2, \cdots, r \). When the station enters the state \( L_i \) a uniformly distributed random backoff counter is chosen from the integer range \( 0, 1, \cdots, W_i - 1 \) where \( W_i \) is the total number of backoff counter values of state \( L_i \).

At a given time slot if the backoff counter of a station is greater than 0 the backoff counter gets decreased by one. Otherwise if the backoff counter reaches 0 the station starts to transmit a packet on a channel for \( m \) time slots with \( m \) as packet duration in time slots. The decision which channel to choose is modeled in Section 2.4. After the \( m \) transmitting time slots the packet got received successfully with probability \( p \) which is assumed to be independent of the state of the station. This is called the decoupling assumption which facilitates the analysis and will be used in the thesis. If the packet got received correctly in state \( L_i \) the station enters the initial state \( L_1 \). Otherwise for \( i < r \) there is a transition to the next state \( L_{i+1} \). This happens with probability \( 1 - p \). For \( i = r \) the station enters the initial state \( L_1 \) independent of transmission success or failure. Given transmission failure at state \( L_r \) we assume the packet is lost.

A station is saturated with packets which means that if the backoff counter of the station reaches 0 in the initial state \( L_1 \) there will be a packet in the queue to transmit. If the transmission fails at state \( L_i \) with \( i < r \) the packet gets retransmitted in the next
state. Otherwise if the station is in state $L_r$ and there occurs a transmission failure the packet is dropped and the station enters $L_1$.

2.3 Model of Receiver

There are $c$ access points acting as receivers and each AP is allocated to one channel. The received signal power depends on the number of transmitting stations on a channel and their transmission power as well as on the locations of the APs and STs. If a given station $ST_a$ with distance $d_{ab}$ to an Access Point $AP_b$ transmits with power $\rho_a$ on the corresponding channel $CH_b$ the received signal to interference and noise ratio at $AP_b$ is defined as

$$\text{SINR}_b = \frac{\rho_a \cdot d_{ab}^{-\gamma}}{N + \sum_{k=1, k \neq a}^{n} \rho_k \cdot d_{kb}^{-\gamma} \cdot \alpha_{jb} \cdot t_{kj}}$$

(2.1)

with $\rho_k$ as transmission power of station $ST_k$, $d_{kb}$ the distance of station $ST_k$ to $AP_b$, $\gamma$ the attenuation exponent, $\alpha_{jb}$ the cross-channel interference factor of channels $CH_j$ and $CH_b$, the indicator variable $t_{kj}$ as

$$t_{kj} = \begin{cases} 1, & \text{if station } ST_k \text{ transmits on channel } CH_j, \\ 0, & \text{otherwise} \end{cases}$$

and $N$ as the noise power. The SINR at an AP has a defined value in every time slot. It is a random variable not only because the noise power is a random variable but also because the number of stations transmitting on a given channel is random due to the backoff-counter which is chosen from a uniform distribution. If $\text{SINR}_b$ is greater than a threshold value $\beta$, i.e.

$$\text{SINR}_b = \frac{\rho_a \cdot d_{ab}^{-\gamma}}{N + \sum_{k=1, k \neq a}^{n} \rho_k \cdot d_{kb}^{-\gamma} \cdot \alpha_{jb} \cdot t_{kj}} > \beta$$

(2.2)
the data sent during the time slot is modeled as being received correctly. A packet is received correctly if Condition 2.2 holds for the $m$ transmitting time slots.

2.4 Models of MAC

In the following sections the considered medium access models are presented.

2.4.1 RARC: Random Access, Random Channel

The model RARC is a random medium access model. If the backoff-counter of a station reaches zero the station chooses one of the $c$ channels randomly and starts to transmit for $m$ time slots on the selected channel independently of the current channel occupation.

2.4.2 RASC: Random Access, Static Channel

Like RARC the MAC model RASC is a random access model. The difference to RARC is that a station can transmit only on one preassigned static channel. If the backoff-counter of a station reaches value zero the station starts to transmit on the preallocated channel for $m$ time slots independently of the current channel occupation.

2.4.3 CSRC: Carrier Sensing, Random Channel

The model CSRC is a carrier sensing model. In this model the stations are assumed to have knowledge about whether there is any other station transmitting but no information about which station is transmitting and which channel is occupied at a given time slot. If a station determines that any station transmits on any channel it pauses its backoff process. Otherwise if the backoff-counter of the station reaches zero the station chooses one of the $c$ channels randomly and starts to transmit for $m$ time slots on the selected channel. When neglecting cross-channel interference and thermal noise a collision may only occur if two stations start to transmit on the same channel at the same time.

2.4.4 CSSC: Carrier Sensing, Static Channel

Like CSRC the MAC model CSSC is a carrier sensing model. The difference to CSRC is that a station can transmit only on one preassigned channel. A station allocated to
channel \( CH_k \) is assumed to have knowledge about whether there is any other station transmitting on channel \( CH_k \). If the station determines that any station is transmitting on channel \( CH_k \) it pauses its backoff process. Otherwise if the backoff-counter of the station reaches zero the station starts to transmit for \( m \) time slots on the preassigned channel. When neglecting cross-channel interference and thermal noise a collision on a channel may only occur if two stations allocated to that channel start to transmit at the same time.

2.5 Models of Interference

So far three types of interference are considered namely the interference of stations transmitting on the same channel as well as cross-channel and noise interference. To simplify the analysis a collision is assumed if two stations transmit on the same channel in a given time slot. In the following further simplifications are presented which are considered in the thesis.

2.5.1 IF: Interference-Free

In the interference free case noise interference as well as cross-channel interference are neglected. Collisions can only occur if two stations transmit on the same channel in a given time slot. The packet duration \( m \) may assume any integer value.

2.5.2 CCNI: Cross-Channel and Noise Interference

In the CCNI model one simplification is done by assuming each station to transmit only for one time slot and therefore \( m = 1 \). Additionally only adjacent channels can interfere. Given a station transmitting on channel \( CH_i \) there occurs a collision if more than two stations transmit on adjacent channels. Otherwise if no other station transmits on channel \( CH_i \) the packet is received successfully. To maintain symmetry circular cross-channel interference is assumed which means that the channel \( CH_1 \) is adjacent to channel \( CH_c \) with \( c \) as total number of channels.

The noise is modeled as chi-square distribution with probability density function

\[
N_{pdf}(x) = \frac{e^{-x/2}}{\sqrt{2\pi x}}
\]

which is the distribution of the square of a standard normal variable.
2. SYSTEM MODEL
Saturation Throughput Analysis

In the following sections the saturation throughput analysis for combinations of the MAC and interference models described in Chapter 2 is conducted. Basic to all considerations is the

Decoupling Assumption 1 The probability $p$ of successful packet transmission given transmission try of a station is independent of the states of all stations.

This assumption was first introduced by Bianchi [4] and has been used extensively by subsequent authors. Its justification is given by the good agreement of simulation and analysis in Chapter 5.

Furthermore to simplify the analysis the product of received transmission power of a station $ST_k$ times the distance attenuation $d_{kb}$ to an Access Point $AP_b$ is assumed to be constant for every pair of station and access point $\rho_0 := \rho_k \cdot d_{kj}^{-\gamma}$ for every $k = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, c$ with $n$ as the total number of stations and $c$ as the total number of channels. This way homogeneity among the stations is given and the value for $p$ can be assumed to be the same for all stations.

Additionally the packet length in bytes $t_p$ is constant and independent of the number of channels. The stations transmission rate $R_c$ is modeled as being proportional to the available bandwidth i.e. $R_c = R_{\text{max}}/c$ with $R_{\text{max}}$ as the maximum transmission rate when the available bandwidth equals the total available bandwidth $B$. Therefore $t_p = m \cdot \sigma_c \cdot R_c = m \cdot \sigma_c \cdot R_{\text{max}}/c \Rightarrow \sigma_c = \frac{t_p \cdot c}{m \cdot R_{\text{max}}}$ holds and the duration of a time slot is a function of the number of channels $c$ and packet duration in time slots $m$.

Table 3.1 lists the most important variables and parameters of this section.
3. SATURATION THROUGHPUT ANALYSIS

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>total number of stations</td>
</tr>
<tr>
<td>$c$</td>
<td>total number of channels</td>
</tr>
<tr>
<td>$m$</td>
<td>packet duration in time slots</td>
</tr>
<tr>
<td>$p_0$</td>
<td>constant product of transmission power and distance attenuation</td>
</tr>
<tr>
<td>$p$</td>
<td>probability of transmission success given transmission try of a packet</td>
</tr>
<tr>
<td>$p^*$</td>
<td>numerically calculated fixed point for $p$</td>
</tr>
<tr>
<td>$p_m$</td>
<td>stationary probability of a station not to transmit for $m$ time slots</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>saturation throughput of a station in bytes per second</td>
</tr>
<tr>
<td>$\tau_{norm}^*$</td>
<td>normalized saturation throughput of a station</td>
</tr>
<tr>
<td>$\tau_{agg}^*$</td>
<td>cumulative saturation throughput of the system</td>
</tr>
<tr>
<td>$t_p$</td>
<td>packet length in bytes</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>stationary attempted transmission probability of a station</td>
</tr>
<tr>
<td>$\lambda_m$</td>
<td>rate of attempted transmissions per time slot</td>
</tr>
<tr>
<td>$\lambda_{0,m}$</td>
<td>rate of successful transmissions per time slot</td>
</tr>
<tr>
<td>$B$</td>
<td>total available bandwidth</td>
</tr>
<tr>
<td>$R_{max}$</td>
<td>maximum transmission rate</td>
</tr>
<tr>
<td>$R_c$</td>
<td>transmission rate with $c$ channels</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>duration of a time slot with $c$ channels</td>
</tr>
</tbody>
</table>

**Table 3.1:** Variables and parameters of the saturation throughput analysis
To perform the saturation throughput analysis it is necessary to calculate the probability of a trip of a station starting and ending in the initial state $L_1$. There are $r$ such trips which are called $tr(i)$ with $i = 1, 2, \cdots, r$. The corresponding probability $\pi(i)$ of a trip given that the station is in the initial state $L_1$ can be derived directly from the Markov chain in Figure 2.1 yielding

$$\pi(i) = \begin{cases} p \cdot (1 - p)^{i-1}, & i < r \\ (1 - p)^{r-1}, & i = r. \end{cases}$$

(3.1)

It equals the product of the probabilities of the state transitions. The sum of the trip probabilities is one $\sum_{i=1}^{r} \pi(i) = 1$. In order to compute the expectation value $E[X]$ of a random variable $X$ the sum over all possible trips starting and ending in state $L_1$ has to be taken

$$E[X] = \sum_{i=1}^{r} \sum_{k=1}^{W_k} \sum_{b_k=0}^{W_k-1} \pi(i) \frac{1}{W_k} \cdot X(b_k, k)$$

$$= \sum_{k=1}^{r} \sum_{i=k}^{W_k-1} \pi(i) \frac{1}{W_k} \cdot X(b_k, k)$$

$$= \sum_{k=1}^{r} \frac{1}{W_k} \sum_{b_k=0}^{W_k-1} X(b_k, k) \cdot \sum_{i=k}^{r} \pi(i)$$

$$= \sum_{k=1}^{r} \frac{1}{W_k} \sum_{b_k=0}^{W_k-1} X(b_k, k) \cdot (1 - p)^{k-1}.$$  (3.2)

In the first step the sum was rearranged. The second step takes out the parts out of the sum which are independent of the summation index. In the last step $\sum_{i=k}^{r} \pi(i) = (1 - p)^{k-1}$ for $k = 1, 2, \cdots, r$ was used.

There are three different saturation throughput values considered namely the saturation throughput in bytes per second $\tau^*$ depending on $n, c, m$ and the transmission rate $R_c = R_{\text{max}}/c$, the normalized saturation throughput and the aggregate or cumulative throughput. The normalized saturation throughput $\tau_{\text{norm}}^*$ is calculated by division by the saturation throughput $\tau^*$ for $n = 1$ and $c = 1$ i.e.

$$\tau_{\text{norm}}^* := \frac{\tau^*(n, c, m, R_{\text{max}})}{\tau^*(1, 1, m, R_{\text{max}})}.$$  (3.3)

Multiplying $\tau_{\text{norm}}^*$ with the total number of stations yields the aggregate throughput

$$\tau_{\text{agg}}^* = n \cdot \tau_{\text{norm}}^* = n \cdot \frac{\tau^*(n, c, m, R_{\text{max}})}{\tau^*(1, 1, m, R_{\text{max}})}.$$  (3.4)
3. SATURATION THROUGHPUT ANALYSIS

The asterisk as superscript indicates a numerically calculated value obtained by a fix point equation.

In the following sections a fix point equation for \( p \) for combinations of the MAC models RARC, RASC, CSRC and CSSC and interference models IF and CCNI is derived and expressions for \( \tau^*, \tau^*_{\text{norm}} \) and \( \tau^*_{\text{agg}} \) in dependence of the fixed point \( p^* \) deduced.

3.1 RARC

3.1.1 IF

In this section the interference-free case is analysed for the MAC model RARC. Thermal noise interference as well as cross-channel interference are neglected.

The saturation throughput analysis is divided into two subproblems a) and b). In a) the quantity \( p = f(p_m) \) which is the probability of successful packet transmission given a transmission try is determined in dependence of probability \( p_m \), which is defined as the stationary probability that a station does not transmit on one given channel for \( m \) time slots. In b) the reverse function for \( p_m = g(p) \) in dependence of \( p \) is derived yielding the fixed point equation \( p = f(g(p)) \) which can be solved numerically.

a) The rigorous analysis of this subproblem is very difficult because the probability \( p_m \) is not independent of the previous \( m \) time slots. However in this section a generalized decoupling assumption is assumed which makes the analysis feasible.

**Generalized Decoupling Assumption 1** The probability \( p_m \) of a station not to transmit during \( m \) time slots is stationary i.e. independent of the states of all stations.

The generalized decoupling assumption can be justified by the good agreement of simulation and analysis in Chapter [5](#).

Since a station always transmits for \( m \) time slots on a randomly chosen channel the probability \( p \) of successful packet transmission given transmission try is the stationary probability that all other stations do not transmit on the same channel for \( m \) time slots

\[
p = p_m^{n-1}. \tag{3.5}
\]

b) \( p_m \) can be derived from the Markov chain in Figure [2.1](#) To that end the expectation value \( E[G_i] \) of time slots of a station spent in each state \( L_i \) with \( i =
3.1 RARC

1, 2, · · · , r is determined where \( G_i \) is a random variable defined by

\[
G_i(b_i) = m + b_i \quad \text{for} \quad b_i = 0, 1, 2, \ldots, W_i - 1.
\]

It is the number of time slots a station spends in state \( L_i \) when the backoff counter \( b_i \) is chosen. An arbitrary but given backoff counter gets selected with probability \( 1/W_i \) because there are \( W_i \) possible uniformly distributed backoff counter values in state \( L_i \). Therefore \( E[G_i] \) is given by

\[
E[G_i] = \sum_{b_i=0}^{W_i-1} P[G_i = b_i] G_i(b_i)
\]

\[
= \sum_{b_i=0}^{W_i-1} \frac{1}{W_i} (m + b_i)
\]

\[
= \frac{1}{W_i} \cdot \sum_{b_i=0}^{W_i-1} (m + b_i)
\]

\[
= m + \frac{W_i - 1}{2} \quad \text{for} \quad i = 1, 2, \ldots, r.
\] (3.6)

Accordingly to 3.2 the mean duration \( G \) of a trip is given by

\[
E[G] = \sum_{k=1}^{r} \left( m + \frac{W_i - 1}{2} \right) \cdot (1 - p)^{k-1}.
\] (3.7)

In the following the expectation value \( E[T_i] \) of a station not to transmit for \( m \) time slots on an arbitrary but given channel \( CH_k \) in state \( L_i \) is determined. The random variable \( T_i \) depends on whether the state \( L_i \) was entered by the station transmitting on channel \( CH_k \) and whether \( CH_k \) is the chosen channel in state \( L_i \). Therefore four cases CaseKN, CaseNN, CaseKK and CaseNK are defined as

CaseKN: The state \( L_i \) was entered by transmission on channel \( CH_k \) and \( CH_k \) is not the transmission channel in state \( L_i \).

CaseNN: The state \( L_i \) was not entered by transmission on channel \( CH_k \) and \( CH_k \) is not the transmission channel in state \( L_i \).

CaseKK: The state \( L_i \) was entered by transmission on channel \( CH_k \) and \( CH_k \) is the transmission channel in state \( L_i \).
3. SATURATION THROUGHPUT ANALYSIS

CaseNK: The state \( L_i \) was not entered by transmission on channel \( CH_k \) and \( CH_k \) is the transmission channel in state \( L_i \).

The probability of selecting an arbitrary but given backoff counter \( b_i \) given one of the four cases is \( \frac{1}{W_i} \) and the stationary probability of the four cases is \( \frac{1}{c^2} \) for CaseKK, \( \frac{c - 1}{c^2} \) for cases CaseNK and CaseKN and \( (\frac{c - 1}{c})^2 \) for CaseNN respectively. The random variable \( T_i \) is defined by

\[
T_i(b_i, \text{Case}) = \begin{cases} 
  b_i + 1, & \text{Case = CaseKN} \\
  b_i, & \text{Case = CaseNK} \\
  m + b_i, & \text{Case = CaseNN} \\
  0, & b_i < m \text{, Case = CaseNN} \\
  b_i - m + 1, & b_i \geq m, \text{Case = CaseKK}.
\end{cases}
\]

It represents the number of time slots a station in state \( L_i \) does not transmit for \( m \) time slots given the backoff counter \( b_i \) and the case Case is chosen. The expectation value of \( T_i \) is determined by

\[
E[T_i] = \sum_{\text{Case}} \sum_{b_i=0}^{b_i=W_i-1} P(T_i(b_i = b_i, \text{Case} = \text{Case})) \cdot T_i(b_i = b_i, \text{Case} = \text{Case})
\]

\[
= P(\text{CaseKN}) \cdot \sum_{b_i=0}^{W_i-1} P(T_i(b_i = b_i, \text{CaseKN}) = b_i) \cdot T_i(b_i = b_i, \text{CaseKN}) \\
+ P(\text{CaseNK}) \cdot \sum_{b_i=0}^{W_i-1} P(T_i(b_i = b_i, \text{CaseNK}) = b_i) \cdot T_i(b_i = b_i, \text{CaseNK}) \\
+ P(\text{CaseNN}) \cdot \sum_{b_i=0}^{W_i-1} P(T_i(b_i = b_i, \text{CaseNN}) = b_i) \cdot T_i(b_i = b_i, \text{CaseNN}) \\
+ P(\text{CaseKK}) \cdot \sum_{b_i=0}^{W_i-1} P(T_i(b_i = b_i, \text{CaseKK}) = b_i) \cdot T_i(b_i = b_i, \text{CaseKK})
\]

\[
= \frac{c - 1}{c^2} \frac{1}{W_i} \sum_{b_i=0}^{W_i-1} (b_i + 1) + \frac{c - 1}{c^2} \frac{1}{W_i} \sum_{b_i=0}^{W_i-1} b_i \\
+ \left(\frac{c - 1}{c}\right)^2 \frac{1}{W_i} \sum_{b_i=0}^{W_i-1} (m + b_i) + \frac{1}{c^2} \frac{1}{W_i} \sum_{b_i=m}^{W_i-1} (b_i - m + 1)
\]

\[
= \frac{1}{c^2} \frac{1}{W_i} \sum_{b_i=m}^{W_i-1} (b_i - m + 1) + \left(\frac{c - 1}{c}\right)^2 \left( m + \frac{W_i - 1}{2} \right) + \frac{c - 1}{c^2} \cdot W_i \quad \text{for } i = 1, 2, \ldots, r.
\]

(3.8)
3.1 RARC

The final result for $p_m$ can be expressed as quotient of expected number of time slots a station does not transmit for $m$ time slots divided by the expected total number of time slots i.e.

$$p_m(p, c, m) = \frac{\sum_{k=1}^{r} E[T_k] \cdot (1 - p)^{k-1}}{\sum_{k=1}^{r} E[G_k] \cdot (1 - p)^{k-1}}. \quad (3.9)$$

Inserting (3.9) into (3.5) yields a fixed point equation for given $n, c, m$

$$p^* = \left( \frac{\sum_{k=1}^{r} E[T_k] \cdot (1 - p^*)^{k-1}}{\sum_{k=1}^{r} E[G_k] \cdot (1 - p^*)^{k-1}} \right)^{n-1} \quad (3.10)$$

which can be solved numerically. Equation (3.10) is the main fixed point equation for the interference free MAC model RARC. From the obtained fixed point $p^*$ the saturation throughput can be calculated. Despite our non trivial assumptions Chapter 5 shows good agreement of theory and simulation and therefore supports Equation (3.10).

For the special case $m = 1$ Equation (3.9) yields

$$p_m(p, c, 1) = \frac{\sum_{k=1}^{r} \left( \frac{1}{c^2} \frac{W_k-1}{2} + \left( \frac{c-1}{c} \right)^2 \left( 1 + \frac{W_k-1}{2} \right) + \frac{c-1}{c} W_k \right) \cdot (1 - p)^{k-1}}{\sum_{k=1}^{r} \left( 1 + \frac{W_k-1}{2} \right) \cdot (1 - p)^{k-1}}$$

$$= \frac{\sum_{k=1}^{r} \left( \frac{W_k}{2} + \frac{1}{c} - \frac{1}{c} \right) \cdot (1 - p)^{k-1}}{\sum_{k=1}^{r} \left( 1 + \frac{W_k-1}{2} \right) \cdot (1 - p)^{k-1}}$$

$$= \frac{\sum_{k=1}^{r} \left( 1 + \frac{W_k-1}{2} - \frac{1}{c} \right) \cdot (1 - p)^{k-1}}{\sum_{k=1}^{r} \left( 1 + \frac{W_k-1}{2} \right) \cdot (1 - p)^{k-1}}$$

$$= 1 - \frac{\lambda}{c}$$

with

$$\lambda := \frac{\sum_{k=1}^{r} (1 - p)^{k-1}}{\sum_{k=1}^{r} \left( 1 + \frac{W_k-1}{2} \right) \cdot (1 - p)^{k-1}}. \quad (3.11)$$

as stationary attempted transmission probability. Equation (3.11) can be derived another way directly from the Markov chain in Figure 2.1 [1], p.7. This suggests Equation (3.9) and therefore (3.10) to be a generalized fixed point equation for $m > 1$.

To get the saturation throughput $\tau^*$ the number of successful transmissions per trip $N_s$ and the number of attempted transmissions per trip $N_a$ are defined. In each state
3. SATURATION THROUGHPUT ANALYSIS

It is $Cost(N_a) = m$ and $Cost(N_s) = p \cdot m$. Then accordingly to 3.2

$$\lambda_m(p) := \frac{E[N_a]}{E[G]} = \frac{\sum_{k=0}^{\infty} m(1 - p)^k}{\sum_{k=0}^{\infty} E[G_{k+1}] \cdot (1 - p)^k}$$

(3.12)

$$\lambda_{0,m}(p) := \frac{E[N_a]}{E[G]} = \frac{\sum_{k=0}^{\infty} m \cdot p(1 - p)^k}{\sum_{k=0}^{\infty} E[G_{k+1}] \cdot (1 - p)^k} = p \cdot \lambda_m(p)$$

(3.13)

hold with $\lambda_m$ as the rate of attempted transmissions per time slot and $\lambda_{0,m}$ as the rate of successful transmissions per time slot. The saturation throughput in bytes per second is then given by

$$\tau^*(n, c, m, R_{\text{max}}) = \lambda_{0,m}(p^*) \cdot \sigma_c \cdot R_{\text{max}} = \lambda_m(p^*) \cdot p^* \cdot \sigma_c \cdot R_{\text{max}}$$

(3.14)

with $p^*$ as the numerically calculated fixed point and $R_{\text{max}}/c$ as the transmission rate. Furthermore $\tau^*$ depends on the total number of stations $n$ and the packet length in time slots $m$.

It is $\tau^*(1, 1, m, R_{\text{max}}) = \frac{m}{m + (W_1 - 1)/2} \cdot R_{\text{max}}$ and therefore the normalized and cumulative throughputs are given by

$$\tau^*_{\text{norm}} = \lambda_m(p^*) \cdot p^* \cdot \frac{m + (W_1 - 1)/2}{m} \cdot \frac{1}{c}$$

(3.15)

and

$$\tau^*_{\text{agg}} = n \cdot \lambda_m(p^*) \cdot p^* \cdot \frac{m + (W_1 - 1)/2}{m} \cdot \frac{1}{c}$$

(3.16)

respectively.

3.1.2 CCNI

In this section the general case with noise and cross-channel interference is analysed for the MAC model RARC for $m = 1$. In the following the quantities $\lambda = f(p)$ in a) and $p = g(\lambda)$ in b) are derived with $\lambda$ as stationary attempted transmission probability of a station in a given time slot and $p$ as probability of transmission success given transmission try.

a) The expression for $\lambda$ as a function of $p$ has been derived implicitly in the previous section. Equation [3.11] can be reused here because the same MAC model is considered as in the interference free case.

b) Let $N_{\text{cdf}}(x)$ be the cumulative density function of the noise power which is modeled as a $\chi^2$ - distribution. There is no closed expression for $N_{\text{cdf}}(x)$. It is defined
as the probability that the noise power at a time slot is less or equal than \( x \). Depending on the cross-channel interference factors \( \alpha_{j} \) in general there can not be an infinite number of stations transmitting on adjacent channels. Let a channel occupation be defined by vector \( v_j \) with \( j = 1, 2, \cdots, c \) and let \( v_j \) denote the number of stations transmitting on channel \( CH_j \). For a given channel occupation let \( s(\vec{v}) \) denote the total number of stations transmitting \( s(\vec{v}) = \sum_{i=1}^{c} v_i \). The channel occupation is a binomial distribution and the stationary probability \( p_{occ} \) of a certain channel occupation \( \vec{v} \) as a function of \( \lambda \) is given by

\[
p_{occ}(\vec{v}) = \binom{n}{s(\vec{v})} \left( \frac{\lambda(p)}{c} \right)^{s(\vec{v})} \left( 1 - \frac{\lambda(p)}{c} \right)^{n-s(\vec{v})}.
\] (3.17)

If \( \text{OCC} \) denotes the set of all possible channel occupations given the total number of stations \( n \) then

\[
p = \sum_{\vec{v} \in \text{OCC}} p_{occ}(\vec{v}) N_{cdf} \left( \frac{\rho_0}{\beta} - \sum_{j=1}^{c} \alpha_{jq} v_j \right)
\] (3.18)

holds where \( q \) can be an arbitrary integer \( q = 1, 2, \cdots, c \) because the sum in (3.18) is taken over all channel occupations. Because noise power assumes only positive values \( N_{cdf}(x) = 0 \) is true for \( x \leq 0 \). Therefore in general the sum in (3.18) reduces drastically to an computable amount.

Given the parameters \( c, n, \beta, \rho_0 \) as well as the \( \alpha_{jq} \) for \( j, q = 1, 2, \cdots, c \) equation (3.18) is only a functions of \( \lambda \) and inserting (3.18) into (3.11) yields a fixed point equation which can be solved numerically.

In the following the cross-channel interference factors are defined to match the model CCNI and equation (3.18) evaluated. Possible parameters at the receiver are \( \rho_0 = 1 \) and \( \beta = 0.9 \) and

\[
\alpha_{ij} = \begin{cases} 
1 & , i = j \\
\frac{1}{2} & , \| i - j \| = 1 \lor (i = 1 \land j = c) \lor (i = c \land j = 1) \\
0 & , \text{otherwise}
\end{cases}
\] (3.19)

for \( i, j = 1, 2, \cdots, c \). If more than two stations transmit on adjacent channels there occurs a collision on the channel. Therefore \( N_{cdf}(x) \) has to be evaluated only for three cases.

Case 1: No other stations transmit on adjacent channels

\[
F_1 := N_{cdf}(10/9) \approx 0.7082.
\]
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Case 2: Only one station transmits on adjacent channels

\[ F_2 := N_{cd}(10/9 - 1/2) \approx 0.5656. \]

Case 3: Two stations transmit on adjacent channels

\[ F_3 := N_{cd}(10/9 - 1) \approx 0.2611. \]

The corresponding stationary probabilities of the three cases are

\[
p_1 := \left(1 - \lambda \frac{3}{c}\right)^{n-1}
\]

\[
p_2 := (n-1)\lambda \frac{2}{c} \left(1 - \lambda \frac{3}{c}\right)^{n-2}
\]

\[
p_3 := (n-1)(n-2)/2 \left(\lambda \frac{2}{c}\right)^2 \left(1 - \lambda \frac{3}{c}\right)^{n-3}. \]

Here \( \lambda \cdot 2/c \) is the stationary probability of a station to transmit on the two adjacent channels and \( 1 - \lambda \cdot 3/c \) represents the stationary probability of a station to transmit on a non-adjacent channel.

Finally we get according to formula 3.18

\[
p = \begin{cases} 
C_1, c = 1 \\
C_2, c = 2 \\
C_3, c > 2 
\end{cases}
\]  

(3.20)

with

\[
C_1 = F_1 \cdot \left(1 - \lambda \frac{1}{c}\right)^{n-1}
\]

\[
C_2 = F_1 \cdot \left(1 - \lambda \frac{2}{c}\right)^{n-1} + F_2 \cdot (n-1)\lambda \frac{2}{c} \left(1 - \lambda \frac{3}{c}\right)^{n-2} + F_3 \cdot \frac{(n-1)(n-2)}{2} \left(\lambda \frac{2}{c}\right)^2 \left(1 - \lambda \frac{3}{c}\right)^{n-3}
\]

\[
C_3 = F_1 \cdot \left(1 - \lambda \frac{3}{c}\right)^{n-1} + F_2 \cdot (n-1)\lambda \frac{2}{c} \left(1 - \lambda \frac{3}{c}\right)^{n-2} + F_3 \cdot \frac{(n-1)(n-2)}{2} \left(\lambda \frac{2}{c}\right)^2 \left(1 - \lambda \frac{3}{c}\right)^{n-3}. \]

Inserting 3.20 into 3.11 yields a fixed point equation which can be solved numerically.

For the saturation throughput analogously to the interference free case

\[
\tau^* = \lambda(p^*) \cdot p^* \cdot \sigma_c \cdot \frac{R_{max}}{c}
\]

(3.21)

\[
\tau_{norm}^* = \frac{\tau^*(n,c, R_{max})}{\tau^*(1,1, R_{max})}
\]

(3.22)

\[
\tau_{agg}^* = n \cdot \tau_{norm}^* = n \cdot \frac{\tau^*(n,c, R_{max})}{\tau^*(1,1, R_{max})}
\]

(3.23)

hold.
3.2 RASC

3.2.1 IF

In this section the interference free case is analysed for the random access and static channel MAC model RASC. To that end each station gets preassigned to one channel on which the station acts. The number of stations on each channel is called \( n_j \) with \( j = 1, 2, \ldots, c \). For the total number of stations \( n = \sum_{j=1}^{c} n_j \) holds and the system is thus divided into \( c \) independent one-channel subsystems of the type analyzed in Section [3.1.1](#). Therefore for each subset of stations \( n_j \) there is in general a different \( p^* \) and \( \tau^* \) and \( \tau^*_\text{norm} \) which accordingly are called \( p^*_j \), \( \tau^*_j \) and \( \tau^*_\text{norm,j} \) respectively. The aggregate throughput is given by

\[
\tau^*_\text{agg} = \sum_{j=1}^{c} n_j \cdot \tau^*_\text{norm,j}.
\] (3.24)

If the total number of stations \( n \) is divisible by the total number of channels \( c \) the system can be divided into \( c \) sets with \( n/c \) stations and each subsystem acting on one channel. Let \( p^*_{\text{RASC+IF}}(n/c, 1, m) \) be the probability of transmission success given transmission try of a station and \( \tau^*_\text{norm, RASC+IF} \) the corresponding normalized saturation throughput. Then the equations

\[
p^* = p^*_{\text{RARC+IF}}(n/c, 1, m)
\] (3.25)
\[
\tau^*_\text{norm} = \tau^*_\text{norm, RARC+IF}(n/c, 1, m)
\] (3.26)
\[
\tau^*_\text{agg} = c \cdot \frac{n}{c} \cdot \tau^*_\text{norm, RARC+IF}(n/c, 1, m)
\]

\[
= n \cdot \tau^*_\text{norm, RARC+IF}(n/c, 1, m)
\] (3.27)

hold where \( p^*_{\text{RARC+IF}}(n/c, 1, m) \) is the probability of transmission success given transmission try of a station in the interference free RARC model with \( n/c \) stations, one channel and \( m \) as packet duration in time slots and \( \tau^*_\text{norm, RARC+IF}(n/c, 1, m) \) the corresponding normalized saturation throughput.

3.2.2 CCNI

In this section the general case with noise and cross-channel interference is analysed for the MAC model RASC for \( m = 1 \). In general there is a different stationary probability of a station to transmit in a given time slot and transmission success probability given
3. SATURATION THROUGHPUT ANALYSIS

transmission try for every subsystem with \( n_j \) number of stations. Therefore in general equations

\[
\lambda_j = \frac{\sum_{k=0}^{r}(1 - p_j)^k}{\sum_{k=0}^{r}(1 - p_j)^k + \sum_{k=0}^{r} \frac{W_j}{2}(1 - p_j)^k}
\]

\[
p_{occ}(\vec{v}) = \prod_{j=1}^{c} \binom{n_j}{v_j} \lambda_j^{v_j} (1 - \lambda_j)^{n_j - v_j}
\]

\[
p_j = \sum_{\vec{v} \in OCC} p_{occ}(\vec{v})N_{cdf} \left( \frac{p_0}{\beta} - \sum_{i=1}^{c} \alpha_{ij} v_i \right)
\]

hold.

If the total number of stations \( n \) is a multiple of the total number of channels \( c \) the stations can be evenly distributed among the channels. Due to symmetry each subsystem of stations has the same \( p_j = p \) for \( j = 1, 2, \cdots, c \). Assuming the interference model CCNI, \( p \) is given by

\[
p = (1 - \lambda)^{n/c - 1} \cdot \begin{cases} 
C_1, & c = 1 \\
C_2, & c = 2 \\
C_3, & c > 2 
\end{cases}
\]

with

\[
C_1 = F_1
\]

\[
C_2 = F_1 \cdot (1 - \lambda)^{n/c} + F_2 \cdot \frac{n}{c}(1 - \lambda)^{n/c - 1} + F_3 \cdot \frac{n/c(n/c + 1)}{2} \lambda^2 (1 - \lambda)^{n/c - 2}
\]

\[
C_3 = F_1 \cdot (1 - \lambda)^{2n/c} + F_2 \cdot \frac{2n}{c} (1 - \lambda)^{2n/c - 1} + F_3 \cdot \frac{2n/c(2n/c + 1)}{2} \lambda^2 (1 - \lambda)^{2n/c - 2}.
\]

The saturation throughput is obtained analogously to the interference free case by \[3.21\] \[3.22\] and \[3.23\]

3.3 CSRC

3.3.1 IF

In this section the interference free CSRC MAC model is analyzed. The idea behind this model is that every time a station transmits the backoff process of each station pauses and the backoff counter does not get decreased. Therefore only the time slots for which the backoff process continues have to be considered which is exactly the case
for the interference free RARC MAC model with \( m = 1 \). A collision occurs only if two stations start transmitting on the same channel. Hence

\[
p = \left(1 - \frac{\lambda}{c}\right)^{n-1}
\]

with \( \lambda \) given by Equation 3.11 holds. However the saturation throughput has to be calculated in a different way because in the previous models there was no pausing of the backoff process. According to [1] \( \tau^* \) can be obtained by the quotient of stationary probability of successful packet transmission \( \lambda \cdot p \) times the amount of data sent per transmission \( \sigma_c \cdot m \cdot R_{\text{max}} / c \) divided by the stationary average real time between counter decrements \( (1 - \lambda)^n \cdot \sigma_c + (1 - (1 - \lambda)^n) \cdot \sigma_c \cdot m \) yielding

\[
\tau^* = \frac{\lambda \cdot p \cdot \sigma_c \cdot m \cdot R_{\text{max}} / c}{(1 - \lambda)^n \cdot \sigma_c + (1 - (1 - \lambda)^n) \cdot \sigma_c \cdot m} = \frac{\lambda \cdot p \cdot m \cdot R_{\text{max}} / c}{(1 - \lambda)^n + (1 - (1 - \lambda)^n) \cdot m}
\]

For the normalized and cumulative saturation throughput

\[
\tau_{\text{norm}}^* = \frac{\tau^*(n, c, m, R_{\text{max}})}{\tau^*(1, 1, m, R_{\text{max}})} \quad (3.32)
\]

\[
\tau_{\text{agg}}^* = n \cdot \frac{\tau^*(n, c, m, R_{\text{max}})}{\tau^*(1, 1, m, R_{\text{max}})} \quad (3.33)
\]

hold.

### 3.3.2 CCNI

This case is omitted. A carrier sensing model for \( m = 1 \) makes no sense. For \( m > 1 \) the analysis is difficult because on the one hand there is the noise power as a random variable per time slot and on the other hand the stationary probability of a station to transmit is not independent per time slot.

### 3.4 CSSC

#### 3.4.1 IF

This section examines the interference free CSSC MAC model. The whole system is divided into \( c \) independent sets with \( n_j \) stations where \( j = 1, 2, \ldots, c \) and \( n = \sum_{j=1}^{c} n_j \). Each subsystem has a different \( \lambda_j, p_j \) and \( \tau_{\text{norm}, j}^* \) which are given by the equations of
3. SATURATION THROUGHPUT ANALYSIS

the RASC MAC model with $c = 1$ namely 3.25 and 3.26. The aggregate saturation throughput is determined by

$$\tau^*_\text{agg} = \sum_{j=1}^{c} n_j \cdot \tau^*_\text{norm,j}. \quad (3.34)$$

3.4.2 CCNI

This case is omitted. A carrier sensing model for $m = 1$ makes no sense. For $m > 1$ the analysis is difficult because on the one hand there is the noise power as a random variable per time slot and on the other hand the stationary probability of a station to transmit is not independent per a time slot.
Simulation

The following section contains a description of the simulation of the saturation throughput used for the MAC models RARC, RASC, CSRC and CSSC with the IF and CCNI interference models respectively. All models use the same state model of the sender. Therefore Section 4.1 describes the basic simulation setup which is the same for all MAC and interference models and Section 4.2 depicts the differences of implementation of the models. Table 4.1 lists the main constants and variables of this section.

4.1 General Description

The main method of the simulation is called nextState() which simulates a time slot. nextState() is structured into three parts Part1, Part2 and Part3 respectively. In the first part new transmitting stations are determined. In the second part the method decides on which channel there occurred a collision and saves the information in wasColl[] and nbrOfColl[]. While wasColl[] stores whether there was a collision in a single time slot nbrOfColl[] saves the information until packet transmission ends. The last part sets the next state of each station. Fig. 4.1 shows the pseudo code of Part3 which is the same for all models. Here state[i][0] stands for the state of the station STi, state[i][1] for the backoff counter and state[i][2] for the number of remaining transmitting time slots given the station transmits.

All stations had a transmission power of one power[i]=1.0 for i = 1, 2, ..., n and a distance to each access point of one distStToAp[i][j]=1.0 for i = 1, 2, ..., n and
## 4. SIMULATION

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>total number of stations</td>
</tr>
<tr>
<td>$c$</td>
<td>total number of channels</td>
</tr>
<tr>
<td>$m$</td>
<td>packet duration in time slots</td>
</tr>
<tr>
<td>$W$</td>
<td>range vector of the backoff counter, $W={32,64,128,256,512}$</td>
</tr>
<tr>
<td>$\text{state}[n][2]$</td>
<td>state of a station $\text{ST}_i$, determined by level $\text{state}[i][0]$, backoff-counter $\text{state}[i][1]$ and remaining number of transmitting time slots $\text{state}[i][2]$</td>
</tr>
<tr>
<td>$\text{chOfSt}[n]$</td>
<td>channel on which the stations transmit</td>
</tr>
<tr>
<td>$\text{distStToAp}[n][c]$</td>
<td>distances of stations to access points</td>
</tr>
<tr>
<td>$\text{crossChIntf}[c]$</td>
<td>cross-channel interference factors</td>
</tr>
<tr>
<td>$\text{wasColl}[c]$</td>
<td>boolean array which contains the information of the collisions on the channels</td>
</tr>
<tr>
<td>$\text{nbrOfColl}[n]$</td>
<td>number of collisions occurred during packet transmission</td>
</tr>
<tr>
<td>$\text{power}[n]$</td>
<td>transmission power of the stations</td>
</tr>
<tr>
<td>$\beta$</td>
<td>SINR-threshold</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>distance attenuation</td>
</tr>
<tr>
<td>$K_{\text{time}}$</td>
<td>counter, counts the number of time slots</td>
</tr>
<tr>
<td>$K_{\text{realtime}}$</td>
<td>counter, counts the total number of time slots passed in the CSRC and CSSC models</td>
</tr>
<tr>
<td>$K_{\text{succ}}$</td>
<td>counter, counts the total number of successful packet transmissions</td>
</tr>
<tr>
<td>$K_{\text{coll}}$</td>
<td>counter, counts the total number of packet collisions</td>
</tr>
</tbody>
</table>

Table 4.1: Variables and constants of the simulation
for $i := 0$ to $n - 1$ step 1 do
    if $(\text{state}[i][1] = 0 \land \text{state}[i][2] > 1)$
        $\text{state}[i][2] := \text{state}[i][2] - 1$;
        comment: decrement number of remaining transmitting time slots
    elsif $(\text{state}[i][1] > 0)$
        $\text{state}[i][1] := \text{state}[i][1] - 1$;
        comment: decrement back-off counter
    elsif $(\text{state}[i][0] = (W.\text{length} - 1) \lor \text{nbrOfColl}[i] < 1)$
        comment: return to initial state
        $\text{state}[i][0] := 0$;
        $\text{state}[i][1] := \text{Random.nextInt}(W[\text{state}[i][0]])$;
        $\text{state}[i][2] := m$;
    else
        comment: collision; go to the next state
        $\text{state}[i][0] := \text{state}[i][0] + 1$;
        $\text{state}[i][1] := \text{Random.nextInt}(W[\text{state}[i][0]])$;
        $\text{state}[i][2] := m$;
    end
end

Figure 4.1: Pseudo code of setting the next station state

$j = 1, 2, \cdots, c$. For all models there was the same $\beta = 0.9$ and $\gamma = 1.0$ respectively.

The backoff counter grows exponentially as can be seen by the range vector $W$. Its length corresponds to the total number of states termed $r$ in the previous chapter.

The counter $K_{\text{succ}}$ counted the total number of successful packet transmissions as well as $K_{\text{coll}}$ the total packet collisions. Additionally there was a counter for the total number of time slots $K_{\text{time}}$. The values for $p^*$ and $\tau^*$ can be obtained by simulation by

\[
p^*_\text{sim} = \frac{K_{\text{succ}}}{K_{\text{succ}} + K_{\text{coll}}} \quad (4.1)
\]

\[
\tau^*_{\text{sim,RA}} = \frac{K_{\text{succ}} \cdot m}{K_{\text{time}} \cdot n} \cdot \frac{R_{\text{max}}}{c} \quad (4.2)
\]

where the subscript $\text{sim}$ indicates that the values where gained from simulation and the subscript $\text{RA}$ the validity of $4.2$ only for the RA MAC models. The relations $4.1$ and $4.2$ are averaged values over all stations.
4. SIMULATION

4.2 Details of Implementation

The differences in the implementation of the simulation of the MAC and interference models lie in Part1 and Part2 of the main method \texttt{nextState()}. While the random channel(RC) and the static channel(SC) MAC differ in Part1 as depicted in Tab 4.2 the interference models IF and CCNI use different cross-channel factors \texttt{crossChIntf[]} and noise power in Part2 to determine a collision at a time slot. The cross-channel interference factors \texttt{crossChIntf[]} refer to cross-channel interference with channel 0. In the interference model IF the noise power is constant and in CCNI it is taken from a chi-square distribution. Tab. 4.3 shows the modeled cross-channel and noise interference in the interference models IF and CCNI.

\begin{table}[h]
\begin{center}
\begin{tabular}{ll}
MAC Model & chOfSt[k] \tabularnewline
\hline
RA & randomly chosen from the integer range 0, 1, \ldots, \(c - 1\) if station starts transmitting \tabularnewline
SC & preassigned to one of the channels 0, 1, \ldots, \(c - 1\) \tabularnewline
\hline
\end{tabular}
\end{center}
\caption{Channel of station \(k\) in the MAC models RA and SC}
\end{table}

\begin{table}[h]
\begin{center}
\begin{tabular}{lll}
Interference Model & \texttt{crossChIntf[i]} & noise \(N\) \tabularnewline
\hline
IF & 1.0 for \(i = 0\) and 0.0 for \(i > 0\) & 1.0 \tabularnewline
CCNI & 1.0 for \(i = 0\) and 0.5 for \(i = 1\), 0.0 for \(i > 1\) & \(\chi^2(x)\) \tabularnewline
\hline
\end{tabular}
\end{center}
\caption{Cross-channel and noise interference in the interference models IF and CCNI}
\end{table}

In the simulation of the carrier sensing models there was an additional counter \(K_{\text{realtime}}\) which counted the total number of time slots passed in each step of \texttt{nextState()}. If at least one station transmitted \(K_{\text{realtime}}\) was increased by \(m\) where \(m\) is the packet duration in time slots. Otherwise \(K_{\text{realtime}}\) was increased by one. Then the saturation throughput \(\tau^*\) was computed as averaged value over all stations

\[ \tau_{\text{sim,CS}}^* = \frac{K_{\text{succ}} \cdot m}{K_{\text{realtime}} \cdot n} \cdot \frac{R_{\text{max}}}{c} \quad (4.3) \]

where the subscript \(CS\) indicates the validity of (4.3) only for the CS MAC models.
5

Comparison Analysis and Simulation

5.1 RARC

5.1.1 IF

In the following section the theoretical and experimental results for successful packet transmission given transmission try $p^*$ as well as the normalized and cumulative throughput $\tau_{\text{norm}}^*$ and $\tau_{\text{agg}}^*$ in dependence of $n$ and $c$ is analysed for the MAC model RARC with interference model IF.

The analytical results $p^*_{\text{ana}}$, $\tau_{\text{norm,ana}}^*$ and $\tau_{\text{agg,ana}}^*$ were numerically calculated from 3.10, 3.15 and 3.16 respectively where the subscript $\text{ana}$ indicates that the values where gained from analysis. Simulation values were obtained from 4.1 and 4.2 respectively.

Fig. 5.1 shows the simulated and calculated values for $p^*$ in dependence of the number of stations $n$ for $m = 15$ and channels $c = 1, 2, \cdots, 20$ where the bottommost line corresponds to $c = 1$ and the greater $c$ the higher the line. Calculated values are pointed out as dots and the simulated values are connected to better distinguish them from the analytical results.

Theoretical and experimental results are in very good agreement with each other. In both cases $p^* = 1$ for $n = 1$ holds independently of $c$ which is reasonable because one single station has no concurrents. Furthermore the trend $\lim_{n \to \infty} p^* = 0$ for every number of channels gets confirmed. For an infinite number of stations the correct packet reception becomes impossible due to the finite number of states of a station.
5. COMPARISON ANALYSIS AND SIMULATION

Figure 5.1: $p^*$ in dependence of $n$ for the RARC MAC with the IF interference model for $m = 15$ and $c = 1, 2, \ldots, 20$. The considered values for $n$ are tick-marked. The dots represent the analytical values. Simulated data points are connected. The bottommost line corresponds to $c = 1$ and the uppermost line to $c = 20$ respectively.
For a given number of stations $p^*$ increases monotonically with the number of channels $c$. This is clear because the probability of a station to select one given channel decreases for greater $c$ resulting in a lower average number of stations transmitting on the channel.

Interestingly enough for a given number of stations $n > 1$ the term $\lim_{c \to \infty} p^*$ is unequal one. Apparently there will always be a collision no matter how many channels there are.

The difference between analytical and calculated values decreases with greater number of channels and stations. Especially for $c$ and $n$ near one their deviation is not negligible. This is shown more clearly in Fig. 5.2 where the part of Fig. 5.1 for $n < 10$ is shown. The reason for the difference is the decoupling approximation which is not valid for small number of channels and stations.

![Figure 5.2](image_url)

**Figure 5.2:** $p^*$ in dependence of $n$ for the RARC MAC with the IF interference model for $m = 15$ and $c = 1, 2 \cdots, 20$. The considered values for $n$ are tick-marked. The dots represent the analytical values. Simulated data points are connected. The bottommost line corresponds to $c = 1$ and the uppermost line to $c = 20$ respectively.

Fig. 5.3 shows the simulated and theoretical values for the normalized throughput $\tau_{\text{norm}}^*$ in dependence of the number of stations $n$ for $m = 15$ and for number of channels.
$c = 1, 3, 10$. The uppermost plot for $n = 1$ corresponds to $c = 1$ and bottommost to $c = 10$.

The throughput decreases exponentially for growing number of stations. If in any of the $m$ transmitting time slots two stations transmit on the same channel there occurs a collision for the entire packet already. However, for small $n$ the optimal number of channels is one whereas for large $n$ the throughput increases with the number of channels. This is shown more clearly in Fig. 5.4 which shows the simulated normalized saturation throughput in dependence of $n$ for $m = 15$ and $c = 1, 2, \ldots, 20$. For large $c$ the term $1/c$ dominates in $3.15$ and therefore $\lim_{c \to \infty} \tau_{\text{norm}}^* = 0$ is true. However for a small total number of channels and large $n$ increasing $c$ yields an improvement of the normalized saturation throughput because the successful transmission probability given transmission try increases to a larger extend than the transmission rate being

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**Figure 5.3:** Simulated and theoretical normalized saturation throughput in dependence of $n$ for the RARC MAC with the IF interference model for $m = 15$ and $c = 1, 3, 10$. Considered values for $n$ are tick-marked. The dots represent the analytical values. Simulated data points are connected. The uppermost line for $n = 1$ corresponds to $c = 1$ and the bottommost line for $n = 1$ to $c = 10$ respectively.
Fig. 5.4: Simulated normalized saturation throughput in dependence of $n$ for the RARC MAC with the IF interference model for $m = 15$, $c = 1, 2, \cdots, 20$. Considered values for $n$ are tick-marked. The uppermost line for $n = 1$ corresponds to $c = 1$ and the bottommost line for $n = 1$ to $c = 20$ respectively.

Fig. 5.5 shows the normalized saturation throughput in dependence of the number of channels for $m = 15$ and the same considered set of stations as in Fig 5.4. The uppermost plot corresponds to $n = 1$ and the bottommost line to $n = 60$. For one station $n = 1$ increasing the number of channels has a negative effect on the throughput because there are no collisions and the transmission rate decreases. If the number of stations is large there are so many collisions that increasing the total number of channels improves the throughput in spite of the transmission rate being lowered. Generally $\lim_{c \to \infty} \tau_{\text{norm}}^* = 0$ holds because the factor $1/c$ dominates in 3.15. Therefore below a threshold value for the number of stations there is an optimal number of channels which maximizes the throughput.

Fig. 5.6 shows the simulated and theoretical values for the aggregation throughput $\tau_{\text{agg}}^*$ in dependence of the number of stations $n$ for $m = 15$ and for number of channels lowered.
Figure 5.5: Simulated and theoretical normalized saturation throughput in dependence of $c$ for the RARC MAC with the IF interference model for $m = 15$ and the same values for $n$ as in Fig. 5.4. The dots represent the analytical values. Simulated data points are connected. The uppermost line corresponds to $n = 1$ and the bottommost line to $n = 60$ respectively.
\( c = 1, 2, 3, 7, 20 \). The uppermost plot for \( n = 1 \) corresponds to \( c = 1 \) and bottommost line to \( c = 20 \). Because \( \tau_{\text{norm}}^* \) is a decreasing function of the number of stations with \( \lim_{n \to \infty} \tau_{\text{norm}}^* = 0 \) and it gets multiplied with the increasing function \( n \) in (3.16) there is a maximum aggregation throughput at a number of stations \( n_{\text{agg}} \). For a small number of channels there is no maximal \( \tau_{\text{agg}}^* \) because the normalized saturation throughput decreases too rapidly. If the number of channels increases \( n_{\text{agg}} \) increases as well and the curve for \( \tau_{\text{agg}}^* \) gets broader. \( \tau_{\text{agg}}^* \) is a measure of the capacity of the system. Therefore for more channels there is more stations maximizing the cumulative throughput.

\[ \begin{align*}
\text{Figure 5.6: } & \text{Simulated and theoretical aggregation throughput in dependence of } n \text{ for the RARC MAC with the IF interference model for } m = 15 \text{ and } c = 1, 2, 3, 7, 20. \text{ Considered values for } n \text{ are tick-marked. The dots represent the analytical values. Simulated data points are connected. The uppermost line for } n = 1 \text{ corresponds to } c = 1 \text{ and the bottommost line for } n = 1 \text{ to } c = 20 \text{ respectively.} \\
\end{align*} \]

In Fig. 5.6, Fig. 5.5 and Fig. 5.3 the theoretical and simulated throughput results are in very good agreement with each other. Only for a total number of stations and channels near 1 there is a not negligible difference due to the decoupling approximation which is not valid anymore in that cases. Therefore the theoretical equations 3.10 3.15
and [3.16] get confirmed by the simulation.

### 5.1.2 CCNI

In the following section the theoretical and experimental results for successful packet transmission given transmission try $p^*$ as well as the normalized and cumulative saturation throughput $\tau_{\text{norm}}$ and $\tau_{\text{agg}}$ in dependence of $n$ and $c$ is analysed for the MAC model RARC with interference model CCNI.

The analytical results $p^*_{\text{ana}}$, $\tau_{\text{norm,ana}}^*$ and $\tau_{\text{agg,ana}}^*$ were numerically calculated from [3.20] and [3.11] for $p^*_{\text{ana}}$ and [3.10], [3.15] and [3.16] for the normalized and cumulative saturation throughput. Simulation results were obtained by [4.1] and [4.2] respectively.

Fig. 5.7 shows the simulated and theoretical values for $p^*$ in dependence of the number of stations $n$ for $m = 15$ and channels $c = 1, 2, \ldots, 10$ where the bottommost line corresponds to $c = 1$ and the greater $c$ the higher the line. Calculated values are pointed out as dots and the simulated values are connected to better distinguish them from the analytical results. Theoretical and experimental results are in very good agreement with each other even for a small number of stations and channels. For $n = 1$ and $c = 1$ there is no guarantee that a packet will be received correctly because of the noise interference. Therefore in that case $p^* \neq 1$ which can also be explained theoretically. For $c = 1$ and $n = 1$ [3.20] yields $p^*(1, 1) = F_1 \approx 0.7082$ which is perfectly compliant with simulation results. For a given number of stations $p^*$ increases monotonically with the number of channels $c$. The probability of a station to select one given channel decreases for greater $c$ resulting in a lower average number of stations transmitting on the channel. In contrast to the interference free case for a given number of stations the term $\lim_{c \to \infty} p^*$ converges to the best achievable value $p^* = F_1$.

Fig. 5.8 shows the simulated and calculated normalized saturation throughput $\tau_{\text{norm}}^*$ in dependence of the number of stations $n$ for and for number of channels $c = 1, 2, \ldots, 10$. The uppermost plot corresponds to $c = 1$ and bottommost line to $c = 10$. For one given station increasing the total number of channels results in a decreasing throughput. In this case the factor $1/c$ in [3.22] dominates. The greater distribution of stations on the channels never achieves a better performance than the scaling factor $c$. Fig. 5.9 shows $\tau_{\text{norm}}^*$ in dependence of the total number of channels $c$ for the set of stations as in Fig. 5.8.
Figure 5.7: Simulated and theoretical values for $p^*$ in dependence of the number of stations $n$ for the RARC MAC with the CCNI interference model for the channels $c = 1, 2, \cdots, 10$ where the bottommost line corresponds to $c = 1$ and the uppermost line to $c = 10$. The considered values for $n$ are tick-marked. The dots represent the analytical values. Simulated data points are connected.
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For a large number of stations $\tau^*_{\text{norm}}$ seems to approach a nonzero value $\lim_{n \to \infty} \tau^*_{\text{norm}} \neq 0$. Since $\lim_{n \to \infty} p^* = 0$ the throughput has to converge to 0 as well. Therefore Fig. 5.8 shows that the throughput converges very slowly for large number of stations.

![Figure 5.8](image)

**Figure 5.8:** Simulated and theoretical normalized saturation throughput in dependence of $n$ for the RARC MAC with the CCNI interference model for $c = 1, 2, \cdots, 10$. Considered values for $n$ are tick-marked. The dots represent the analytical values. Simulated data points are connected. The uppermost line corresponds to $c = 1$ and the bottommost line to $c = 10$ respectively.

Fig. 5.10 shows the simulated and theoretical values for the aggregation throughput $\tau^*_{\text{agg}}$ in dependence of the number of stations $n$ and for the number of channels $c = 1, 2, 3, 7$. The uppermost plot corresponds to $c = 1$ and bottommost line to $c = 7$. Since $\lim_{n \to \infty} \tau^*_{\text{norm}} = 0$ there has to be a number of stations $n_{\text{agg}}$ which maximizes $\tau^*_{\text{agg}}$. The slow convergence of the normalized saturation throughput causes the cumulative throughput to increase for a small total number of stations. In Fig 5.6
Figure 5.9: Simulated and theoretical normalized saturation throughput in dependence of $c$ for the RARC MAC with the CCNI interference model for the set of stations as in 5.8. Dots are the analytical values. Simulated data points are connected. The uppermost line corresponds to $n = 1$ and the bottommost line to $n = 60$ respectively.
Figure 5.10: Simulated and theoretical aggregation throughput for the RARC MAC with the CCNI interference model in dependence of the number of stations for channels 1, 2, 3 and 7. The uppermost plot corresponds to \( c = 1 \) and bottommost plot to \( c = 7 \). The analytical values are plotted as dots. Simulated data points are connected. Considered values for \( n \) are tick-marked.
5.2 RASC

5.2.1 IF

In the following section the theoretical and experimental results for successful packet transmission given transmission try \( p^* \) as well as the normalized and cumulative throughput \( \tau^{\ast}_{\text{norm}} \) and \( \tau^{\ast}_{\text{agg}} \) in dependence of \( n \) and \( c \) is analysed for the MAC model RASC with interference model IF.

Fig. 5.11 shows \( p^* \) in dependence of \( n \) for channels \( c = 1, 2 \cdots, 6 \). The number of stations was chosen in order to be a multiple of the total number of channels and the stations could get distributed evenly on the channels. The analysis was performed with Equations 3.25, 3.26 and 3.27. Simulation results were obtained by 4.1 and 4.2 respectively.

For a given \( n \) increasing \( c \) results in larger \( p^* \). Theoretical and experimental results

Figure 5.11: \( p^* \) in dependence of \( n \) for the RASC MAC with the IF interference model for \( m = 15, c = 1, 2 \cdots, 6 \). The considered values for \( n \) are chosen to be a multiple of \( c \). The dots represent the analytical values. Simulated data points are connected. The bottommost line corresponds to \( c = 1 \) and the uppermost line to \( c = 6 \) respectively.
agree only for \( n = 1 \) and a large number of stations. For a small number of stations on one channel the decoupling approximation is not valid any more as seen in Fig. 5.2. The error for a small system gets multiplied by \( c \) for the whole system.

Fig. 5.12 and 5.13 show the simulated and calculated saturation throughput in dependence of \( n \) for \( m = 15 \). Theoretical and experimental results differ significantly for a small quotient \( n/c \). Fig. 5.13 shows a zoomed in part of Fig. 5.12. Theory and analysis differ significantly for \( n/c < 5 \). The normalised saturation throughput decreases exponentially due to the large packet duration in time slots \( m = 15 \). In the contrast to the RARC model for a given \( n \) the normalized saturation throughput increases when the total number of channels gets increased. This behaviour can be explained by the low performance of the system with small number of channels.

Figure 5.12: Simulated and theoretical normalized saturation throughput in dependence of \( n \) for the RASC MAC with the IF interference model for \( m = 15 \) and \( c = 1, 2, \ldots, 6 \). The values for \( n \) are chosen to be a multiple of \( c \). The dots represent the analytical values. Simulated data points are connected. The bottommost line corresponds to \( c = 1 \) and the uppermost line to \( c = 6 \) respectively.

Fig. 5.14 shows the aggregation throughput for the RASC MAC and IF interference model. The term \( 1/c \) dominates in 3.26 even when the normalized saturation
Figure 5.13: Simulated and theoretical normalized saturation throughput in dependence of \( n \) for the RASC MAC with the IF interference model for \( m = 15 \) and \( c = 1, 2, \ldots, 6 \). The values for \( n \) are chosen to be a multiple of \( c \). The dots represent the analytical values. Simulated data points are connected. The bottommost line corresponds to \( c = 1 \) and the uppermost line to \( c = 6 \) respectively.
throughput gets multiplied with the increasing function $n$.

Figure 5.14: Simulated and theoretical aggregation throughput in dependence of $n$ for the RASC MAC with the IF interference model for $m = 15$ and $c = 1, 2, \cdots, 6$. The values for $n$ are chosen to be a multiple of $c$. The dots represent the analytical values. Simulated data points are connected. The bottommost line corresponds to $c = 1$ and the uppermost line to $c = 6$ respectively.

5.2.2 CCNI

In this section the theoretical and experimental results for $p^*$, $\tau_{\text{norm}}^*$ and $\tau_{\text{agg}}^*$ for the RASC MAC with the CCNI interference model are discussed. The calculated values are obtained from 3.29 and 3.28 for $p^*$ and from 3.14 3.3 and 3.4 for the normalized and cumulative saturation throughput. Simulation result are gained from 4.1 and 4.2 respectively.

Fig. 5.15 shows the theoretical and experimental transmission success given transmission try probability $p^*$ in dependence of the total number of stations $n$ for the channels $c = 1, 2, \cdots, 6$. As expected increasing $n$ for a given $c$ lowers $p^*$ because on average there are more stations operating on one channel. Increasing $c$ for a given $n$ increases
5.2 RASC

$p^*$ because on average there are less stations operating on one channel. The maximum $p^*$ is achieved for $n = 1$ and $c = 1$. According to (3.29) this value is $p^*_{\text{max}} = F_1 \approx 0.7082$ which is compliant with simulation. Fig. 5.15 also suggests $\lim_{c \to \infty} p^* = F_1$. Theory and simulation are in very good agreement with each other which confirms equation (3.29).

\[ F_1 \approx 0.7082 \]

Fig. 5.15: $p^*$ in dependence of $n$ for the RASC MAC with the CCNI interference model for $c = 1, 2, \cdots, 6$. The considered values for $n$ are chosen to be a multiple of $c$. The dots represent the analytical values. Simulated data points are connected. The bottommost line corresponds to $c = 1$ and the uppermost line to $c = 6$ respectively.

Fig. 5.16 shows the theoretical and experimental normalized saturation throughput in dependence of the total number of stations $n$ for the channels $c = 1, 2, \cdots, 6$. Increasing $n$ results in a decreasing $\tau^*_{\text{norm}}$ because $p^*$ gets decreased. Increasing $c$ lowers $\tau^*_{\text{norm}}$ as well. When $c$ gets increased the rise of the probability $p^*$ of transmission success given transmission try cannot compensate the scaling factor $1/c$ in (3.3) which therefore dominates the saturation throughput. Interestingly enough Fig. 5.16 suggests $\lim_{n \to \infty} \tau^*_{\text{norm}} > 0$. Due to $\lim_{n \to \infty} p^* = 0$ for a large number of stations the saturation throughput converges very slowly to 0. Theory and simulation are in very good agreement.
agreement with each other which confirms the relations \(3.14\) \(3.3\) for this model.

Figure 5.16: Simulated and theoretical normalized saturation throughput in dependence of \(n\) for the RASC MAC with the CCNI interference model for \(c = 1, 2, \cdots, 6\). The values for \(n\) are chosen to be a multiple of \(c\). The dots represent the analytical values. Simulated data points are connected. The uppermost line corresponds to \(c = 1\) and the bottommost line to \(c = 6\) respectively.

Fig. 5.17 shows the theoretical and experimental aggregation throughput in dependence of the total number of stations \(n\) for the channels \(c = 1, 2, \cdots, 6\). The slow convergence of the normalized saturation throughput causes the cumulative throughput to increase for a small total number of stations. Theory and simulation are in very good agreement with each other which confirms the relation \(3.4\) for this model.

5.3 CSRC

5.3.1 IF

In this section the theoretical and experimental results for \(\tau_{\text{norm}}^*\) and \(\tau_{\text{agg}}^*\) for the CSRC MAC with the IF interference model are discussed. The theoretical values for the
Figure 5.17: Simulated and theoretical aggregation throughput in dependence of $n$ for the RASC MAC with the CCNI interference model for $c = 1, 2, \cdots, 6$. The values for $n$ are chosen to be a multiple of $c$. The dots represent the analytical values. Simulated data points are connected. The uppermost line corresponds to $c = 1$ and the bottommost line to $c = 6$ respectively.
normalized and cumulative saturation throughput were obtained from 3.31, 3.32 and 3.33 respectively. Simulation results were calculated by the average expression 4.3.

Fig. 5.18 shows the theoretical and experimental normalized saturation throughput in dependence of the total number of stations $n$ for the channels $c = 1, 2, \cdots, 10$. Increasing $n$ results in a decreasing $\tau_{norm}^*$ because the probability $p^*$ of transmission success given transmission try gets decreased. Increasing $c$ lowers $\tau_{norm}^*$ as well. When $c$ gets increased the rise of $p^*$ cannot compensate the scaling factor $1/c$ in 4.3 which therefore dominates the saturation throughput. $\lim_{n \to \infty} \tau_{norm}^* = 0$ holds as expected because $\lim_{n \to \infty} p^* = 0$. Theory and simulation are in very good agreement with each other which confirms the relation 3.31 for this model.

Figure 5.18: Simulated and theoretical normalized saturation throughput in dependence of $n$ for the CSRC MAC with the IF interference model for $m = 15$ and $c = 1, 2, \cdots, 10$. The considered values for $n$ are tick-marked. The dots represent the analytical values. Simulated data points are connected. The uppermost line corresponds to $c = 1$ and the bottommost line to $c = 10$ respectively.

Fig. 5.19 shows the theoretical and experimental aggregation throughput in dependence of the total number of stations $n$ for the channels $c = 1, 2, \cdots, 10$. There is a
number of stations $n_{agg}$ which maximizes the aggregation throughput. For large $n$ the cumulative as well as the normalized throughput converge slowly to 0. Theory and simulation are in very good agreement with each other which is a result from the good agreement of the normalized saturation throughput $\tau_{norm}^\ast$.

Figure 5.19: Simulated and theoretical aggregation throughput in dependence of $n$ for the CSRC MAC with the IF interference model for $m = 15$ and $c = 1, 2, \ldots, 10$. The considered values for $n$ are tick-marked. The dots represent the analytical values. Simulated data points are connected. The uppermost line corresponds to $c = 1$ and the bottommost line to $c = 10$ respectively.

5.4 CSSC

5.4.1 IF

In this section the theoretical and experimental results for $\tau_{norm}^\ast$ and $\tau_{agg}^\ast$ for the CSSC MAC with the IF interference model are discussed. The theoretical values for the normalized saturation throughput were obtained from [3.26] with $c = 1$ and for the aggregate saturation throughput from [3.34] respectively. Simulation results were calculated by the
average expression \[ 4.3 \]

Fig. 5.20 shows the theoretical and experimental normalized saturation throughput in dependence of the total number of stations \( n \) for the channels \( c = 1, 2, \cdots, 10 \). For a number of stations \( n < 10 \) the saturation throughput decreases with the number of channels \( c \) due to the scaling factor \( 1/c \) in the transmission rate. However for \( n > 10 \) the increasing number of channels \( c \) implies a lower number of station per channel and therefore an increasing transmission success probability given transmission try of a station. Both effects compensate which results in an equal normalized saturation throughput for a large number of stations.

**Figure 5.20:** Simulated and theoretical normalized saturation throughput in dependence of \( n \) for the CSSC MAC with the IF interference model for \( m = 15 \) and \( c = 1, 2, \cdots, 10 \). The considered values for \( n \) are tick-marked. The dots represent the analytical values. Simulated data points are connected. The uppermost line corresponds to \( c = 1 \) and the bottommost line to \( c = 10 \) respectively.

Fig. 5.21 shows the theoretical and experimental aggregation throughput in dependence of the total number of stations \( n \) for the channels \( c = 1, 2, \cdots, 10 \). There is a number of stations \( n_{agg} \) which maximizes the aggregation throughput. For large \( n \) the
aggregation throughput increases with increasing number of channels $c$. This shows that the normalized saturation throughput for a large number of stations is slightly greater for a greater number of channels due to the effect of less stations per channel. Theory and simulation are in very good agreement with each other which is a result from the good agreement of the normalized saturation throughput $\tau_{\text{norm}}^*$. 

**Figure 5.21:** Simulated and theoretical aggregation throughput in dependence of $n$ for the CSSC MAC with the IF interference model for $m = 15$ and $c = 1, 2, \cdots, 10$. The considered values for $n$ are tick-marked. The dots represent the analytical values. Simulated data points are connected. The bottommost line for large $n$ corresponds to $c = 1$ and the uppermost line for large $n$ to $c = 10$ respectively.
5. COMPARISON ANALYSIS AND SIMULATION
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Summary and Discussion

In this thesis the saturation throughput has been analysed for the MAC models RARC, RASC, CSRC and CSSC. In the random access models RARC and RASC both the interference free case and the case with cross-channel and noise interference were considered. For the carrier sensing MAC models CSRC and CSSC only the interference free case was evaluated. The analysis yielded expressions for the saturation throughput which could be calculated numerically by a fix point equation. The normalized saturation throughput was plotted in dependence of the system parameters $n$ as total number of stations and $c$ as total number of channels. A simulation was implemented to assess the saturation throughput for all model combinations.

The results for the saturation throughput obtained from simulation and analysis were in good agreement with each other. Therefore the decoupling assumptions made to make the analysis feasible are justified and the theoretically deduced equations confirmed. For a small number of stations and channels the theoretical and simulated saturation throughput values differed significantly which can be explained by the decoupling approximation which is no longer valid in that case.

For all models the expected trend of decreasing saturation throughput with increasing number of stations gets confirmed. The dependence of the saturation throughput of the number of channels in not trivial. For the interference free RARC MAC model for some number of stations there exists an optimal number of channels which maximizes the saturation throughput. This can be explained by the bad performance of the system for small number of channels. All other models show a decreasing saturation throughput when the number of channels gets increased due to the lower transmission
rate for bigger number of channels. However for the cross-channel and noise interference MAC models RARC and RASC the prediction of the existence of an optimal number of channels can be made when the system parameters expose high cross-channel and noise interference.

A comparison of the MAC models can be done for the interference free and for the cross-channel and noise interference model. For the CCNI model the normalized saturation throughput of the MAC models RARC and RASC in Fig. 5.8 and Fig. 5.16 can be compared. The normalized saturation throughput is equal for every number of stations and channels for both models. On average it does not matter whether the station selects an arbitrary channel randomly or gets preassigned to one channel beforehand for these models.

For the IF interference model the normalized saturation throughput of all MAC models RARC in Fig. 5.3, RASC in Fig. 5.12, CSRC in Fig. 5.18 and CSSC in Fig. 5.20 can be compared. The normalised saturation throughput of the CSSC MAC model shows very low dependence of the number of channels while in the CSRC model the saturation throughput decreases with increasing number of channels. Therefore for the carrier sensing models it is better to have the stations preassigned to one channel beforehand. This statement it also true for the random access MAC models which can be seen by comparing Fig. 5.3 and Fig. 5.12. Comparing the random access and the carrier sensing models in general the carrier sensing models expose a better performance. However increasing the total number of channels results in a decreasing normalised saturation throughput in the CSRC MAC model and in an increasing normalised saturation throughput in the RASC MAC model. Therefore above a threshold for the number of channels the model RASC will show a better performance than the CSRC MAC model.

The thesis concludes with a remark on the derived theoretical expressions for the probability \( p^* \) of transmission try given transmission success and for the saturation throughputs. The results obtained by analysis and simulation show aside for a small number of stations very good agreement with each other as discussed in the previous chapter. On the other hand the derived equations are very powerful because they depend on the freely selectable system parameters \( n \) as total number of stations, \( c \) as total number of channels, \( m \) as packet duration in time slots and \( W \) as the back-off counter vector from which the uniformly distributed random back-off counter gets
selected. Additionally expressions for arbitrary cross-channel interference factors and an arbitrary noise distribution can be easily derived. Therefore much more theoretical analysis of the system properties than has been done in this thesis can be conducted.
6. SUMMARY AND DISCUSSION
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