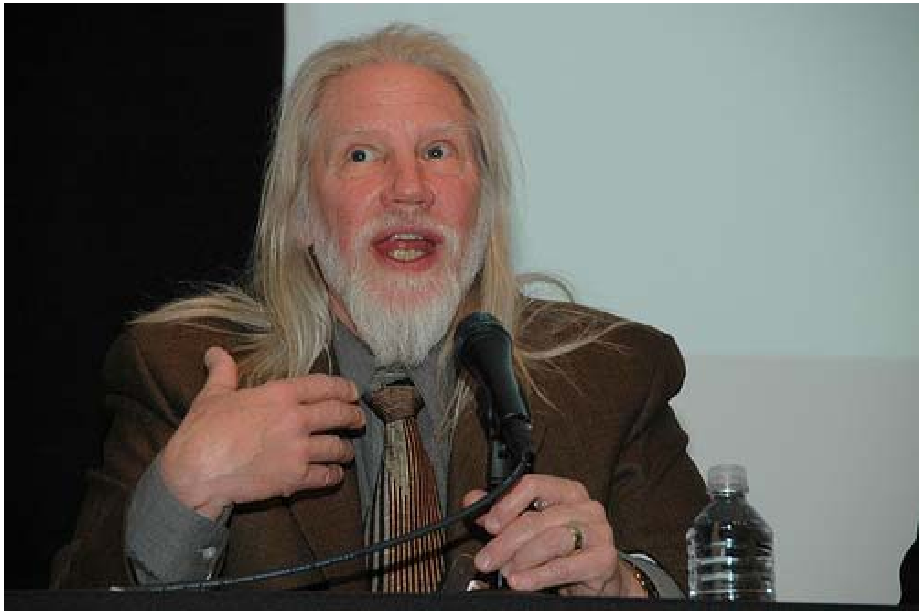


Crypto Basics

Recent block cipher: AES

Public Key Cryptography

Public key exchange: Diffie-Hellmann

Homework suggestion

What is a cryptosystem?

- $K = \{0,1\}^l$
- $P = \{0,1\}^m$
- $C' = \{0,1\}^n$, $C \subseteq C'$

- $E: P \times K \rightarrow C$
- $D: C \times K \rightarrow P$

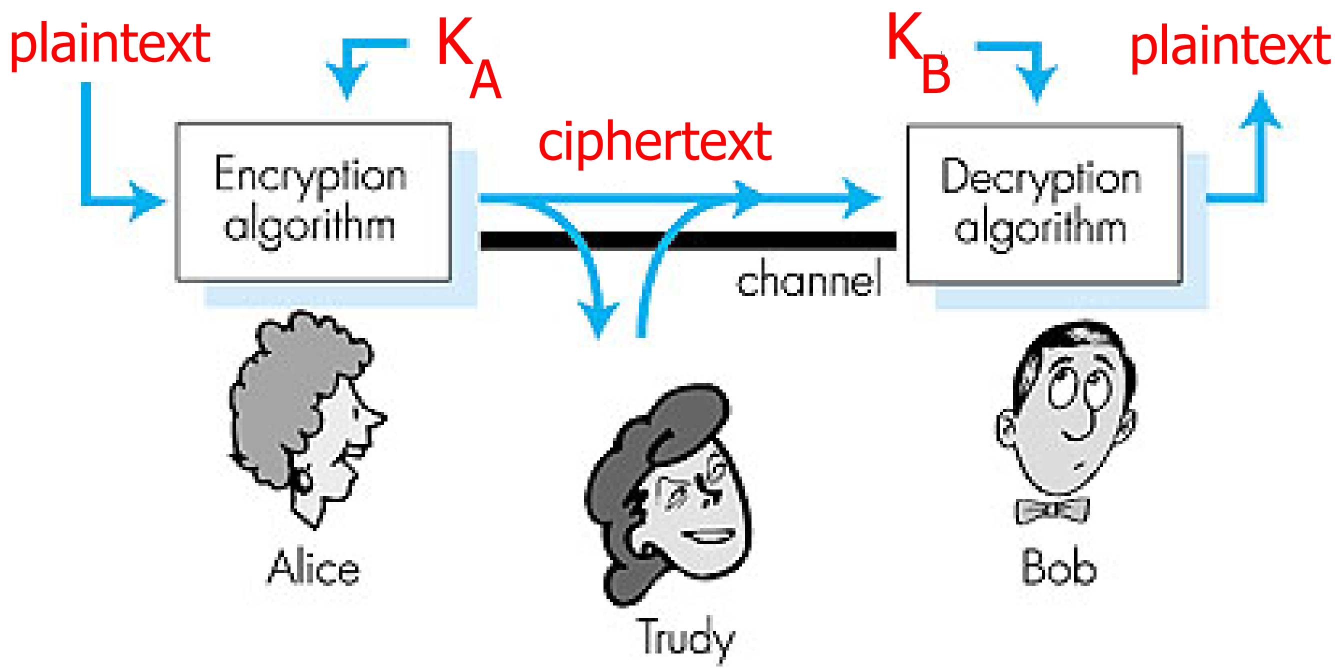
- $\forall\ p \in P, k \in K: D(E(p,k),k) = p$
    - It is *infeasible* to find inversion $F: P \times C \rightarrow K$

Lets start again!
This time in English … .

What is a cryptosystem?

- A pair of algorithms that take a key and convert plaintexts to ciphertexts and backwards later
  - Plaintext: text to be protected
  - Ciphertext: should appear like random
- Requires sophisticated math!
  - Do not try to design your own algorithms!

The language of cryptography



- Symmetric or secret key crypto:
  sender and receiver keys are identical and ***secret***

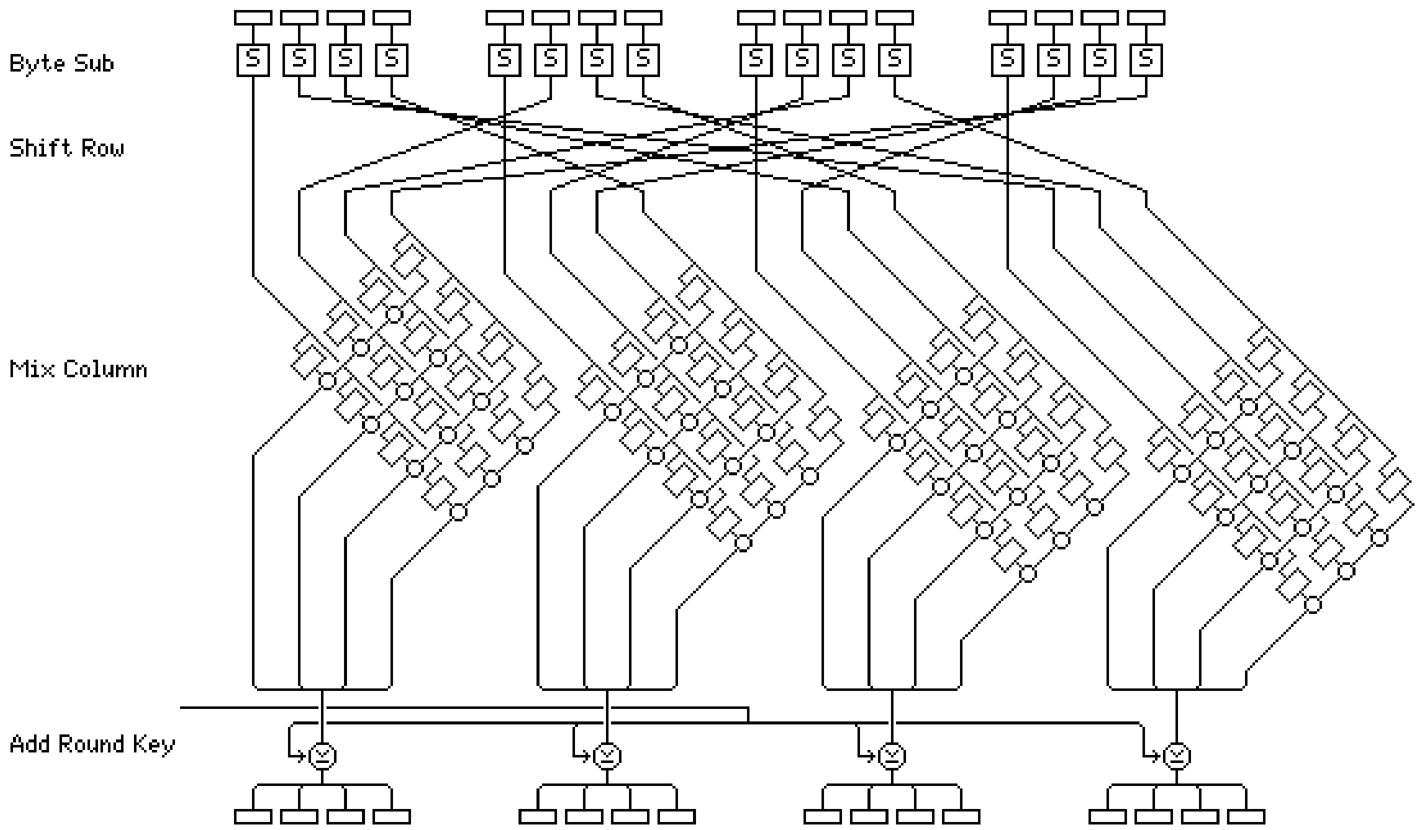
- Asymmetric or Public-key crypto:
  encrypt key public, decrypt key secret

Strength of DES???

- 56-bit keys have $2^{56} = 7.2 \times 10^{16}$ values
- Brute force search looked hard in the seventies/eighties
- Recent advances have shown is possible
  - in 1997 on Internet in a few months
  - in 1998 on dedicated h/w (EFF) in a few days
  - in 1999 above combined in 22hrs!
- Now have several analytic attacks on DES
  - these utilise some deep structure of the cipher
    - by gathering information about encryptions
    - can eventually recover some/all of the sub-key bits
    - if necessary then exhaustively search for the rest
- Generally these are statistical attacks
- Include
  - differential cryptanalysis
  - linear cryptanalysis
  - related key attacks

- Thus, consider alternatives to DES – AES

AES – Advanced Encryption Standard



Origins

- A replacement for DES was needed
  - have theoretical attacks that can break it
  - have demonstrated exhaustive key search attacks
- Triple-DES possible – but slow, has small blocks
- US NIST issued a call for ciphers in 1997
- 15 candidates accepted in Jun 98
- 5 were shortlisted in Aug-99
- Rijndael was selected as the AES in Oct-2000
- Accepted as FIPS PUB 197 standard in Nov-2001

AES Requirements

- Private key symmetric block cipher
- 128-bit data, 128/192/256-bit keys
- Stronger & faster than Triple-DES
- Active life of 20-30 years (+ archival use)
- Full open specification & design details
- Good C, Java, and hw implementations
- NIST releases all submissions & unclassified analyses

AES Evaluation Criteria

- Initial criteria:
  - security – effort for practical cryptanalysis
  - cost – in terms of computational efficiency
  - algorithm & implementation characteristics
- Final criteria
  - general security
  - ease of software & hardware implementation
  - implementation attacks
  - flexibility (in en/decrypt, keying, other factors)

AES Shortlist

- After testing and evaluation, shortlist in Aug-99:
  - MARS (IBM) - complex, fast, high security margin
  - RC6 (USA) - v. simple, v. fast, low security margin
  - Rijndael (Belgium) - clean, fast, good security margin
  - Serpent (Euro) - slow, clean, v. high security margin
  - Twofish (USA) - complex, v. fast, high security margin
- Then subject to further analysis & comment
- Saw contrast between algorithms with
  - few complex rounds verses many simple rounds
  - which refined existing ciphers vs. new proposals

The AES Cipher - Rijndael

- Designed by Rijmen-Daemen in Belgium
- Has 128/192/256 bit keys, 128 bit data or block size
- An **iterative** rather than **feistel** cipher
  - processes data as block of 4 columns of 4 bytes
  - operates on entire data block in every round
- Designed to be:
  - resistant against known attacks
  - speed and code compactness on many CPUs
  - design simplicity

Rijndael

input bytes

in_0	in_4	in_8	in_{12}
in_1	in_5	in_9	in_{13}
in_2	in_6	in_{10}	in_{14}
in_3	in_7	in_{11}	in_{15}

→

State array

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,2}$	$s_{3,3}$

→

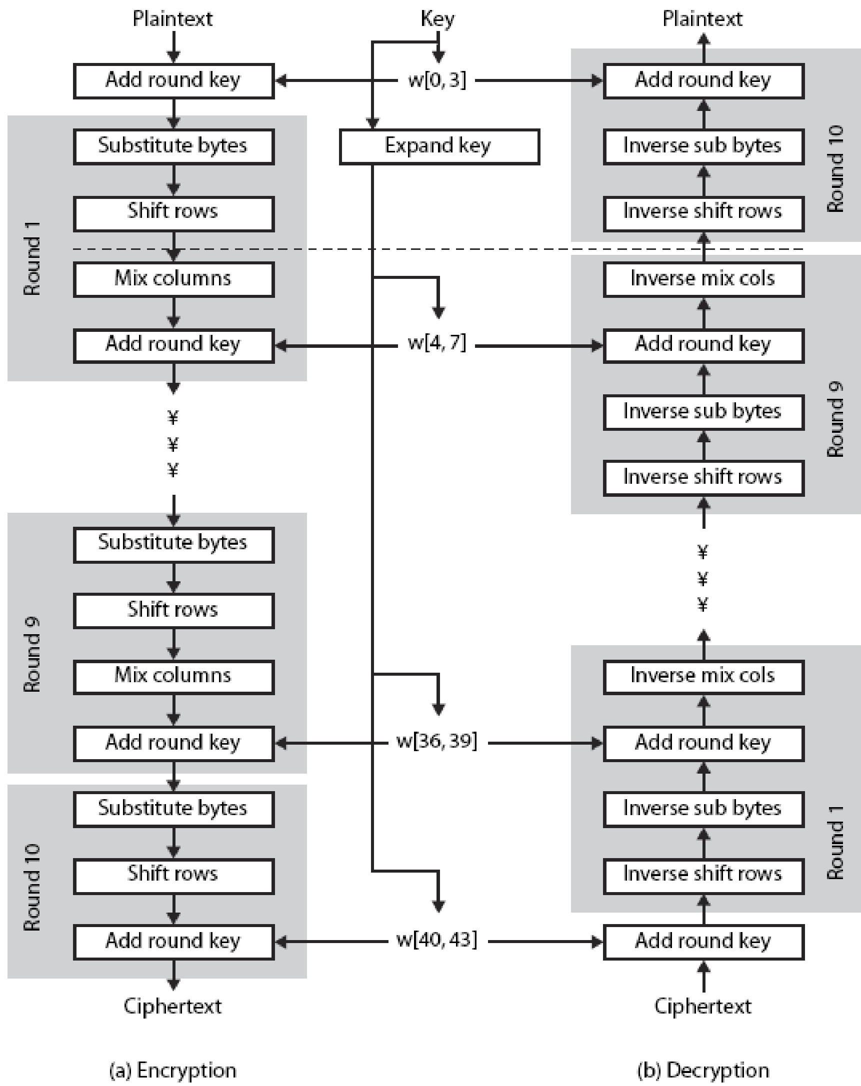
output bytes

out_0	out_4	out_8	out_{12}
out_1	out_5	out_9	out_{13}
out_2	out_6	out_{10}	out_{14}
out_3	out_7	out_{11}	out_{15}

Figure 3. State array input and output.

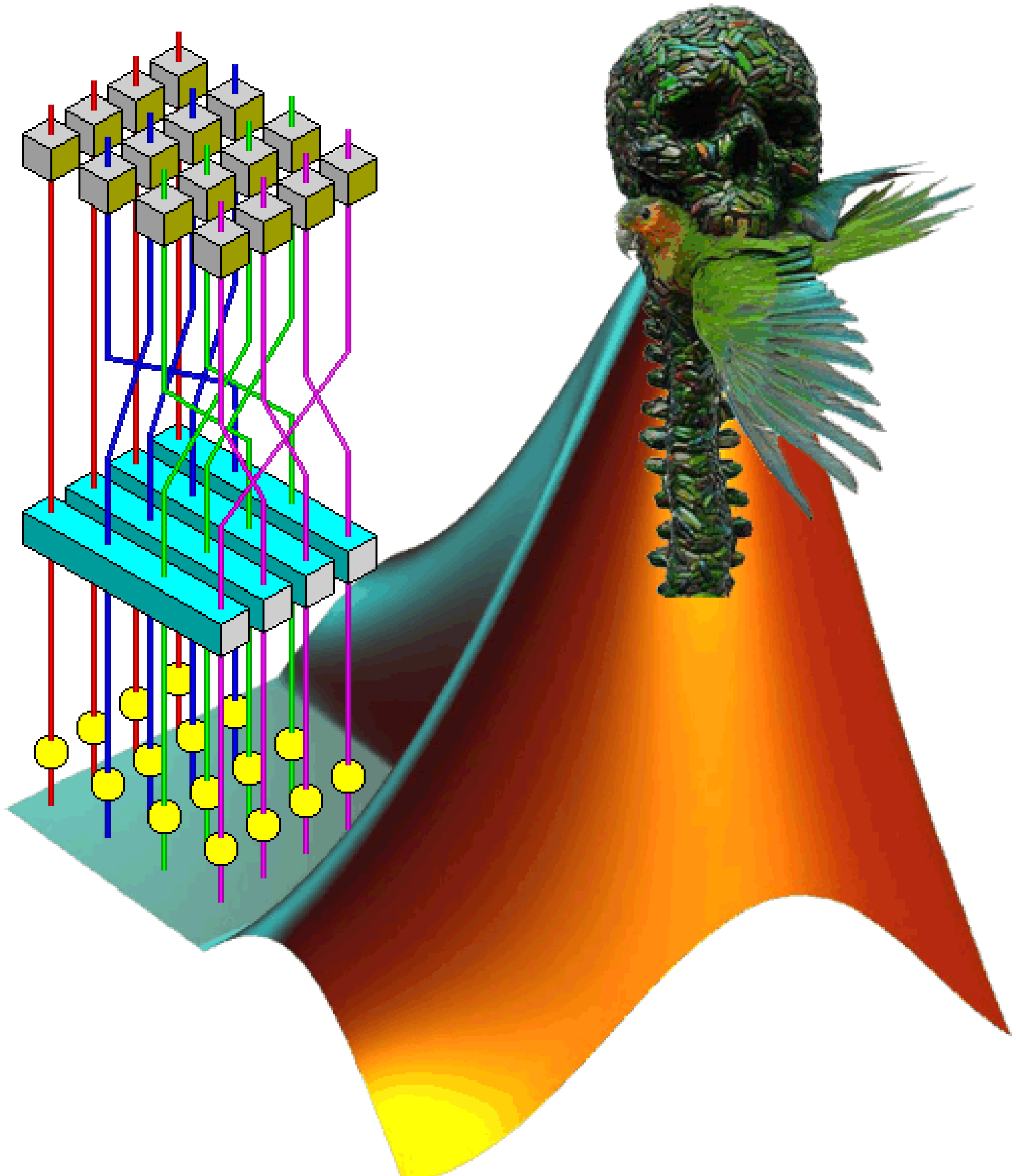
- Data block of 4 columns of 4 bytes is state
- Key is expanded to array of words
- Has 9/11/13 rounds in which state undergoes:
  - byte substitution (1 S-box used on every byte)
  - shift rows (permute bytes between groups/columns)
  - mix columns (subs using matrix multipy of groups)
  - add round key (XOR state with key material)
  - view as alternating XOR key & scramble data bytes
- Initial XOR key material & incomplete last round
- With fast XOR & table lookup implementation

Rijndael



Plaintext
Key
Plaintext
Add round key
w[0, 3]
Add round key
Round 1
Substitute bytes
Expand key
Inverse sub bytes
Round 10
Shift rows
Inverse shift rows
Mix columns
Inverse mix cols
Add round key
w[4, 7]
Add round key
Inverse sub bytes
Round 9
Inverse shift rows
¥
¥
¥
Round 9
Substitute bytes
Shift rows
Mix columns
Inverse mix cols
Add round key
w[36, 39]
Add round key
Round 1
Inverse sub bytes
Round 10
Substitute bytes
Shift rows
Inverse shift rows
Add round key
w[40, 43]
Add round key
Ciphertext
Ciphertext
(a) Encryption
(b) Decryption

Rijndael

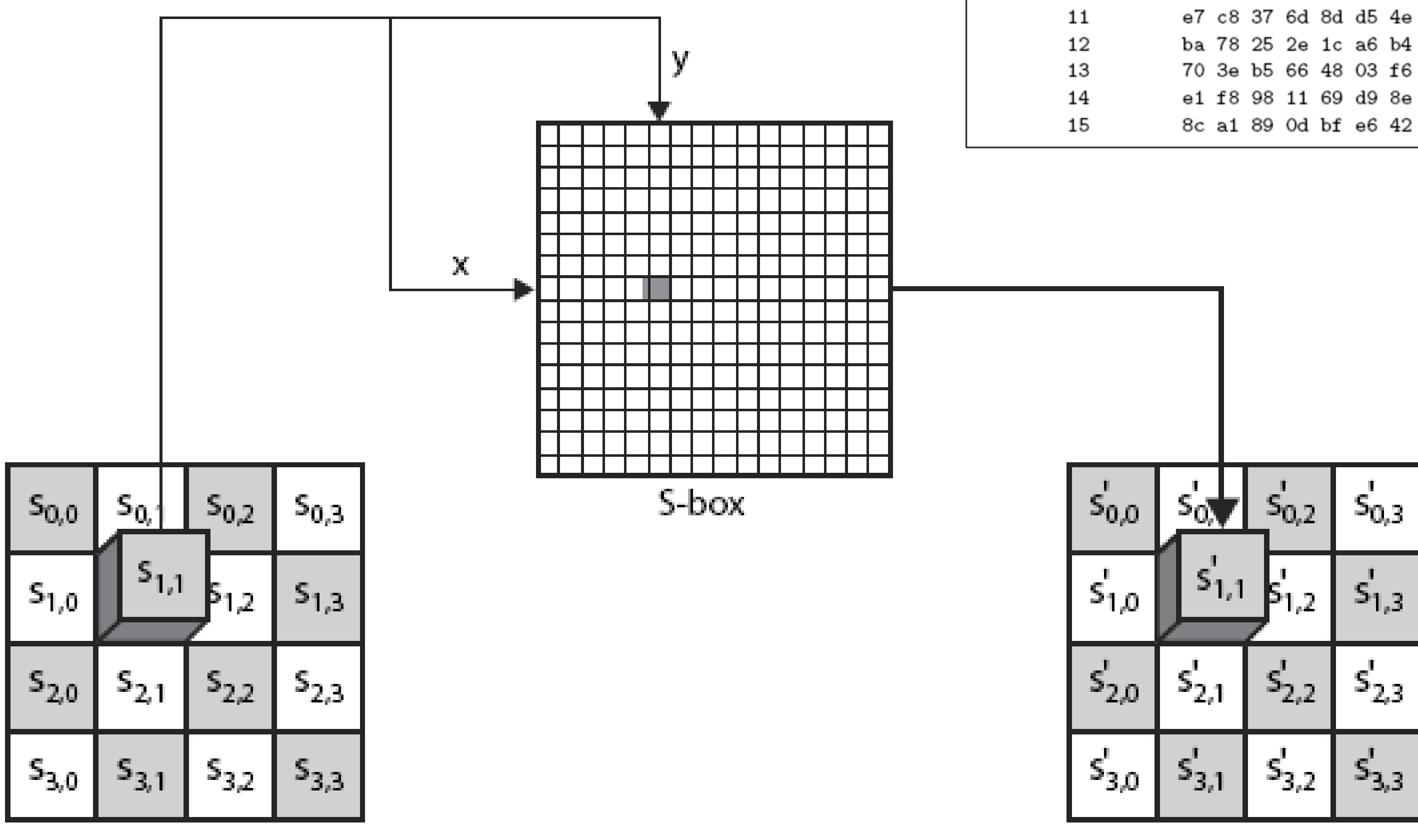


Byte Substitution

- a simple substitution of each byte
- uses one table of 16x16 bytes containing a permutation of all 256 8-bit values
- each byte of state is replaced by byte indexed by row (left 4-bits) & column (right 4-bits)
  - eg. byte {95} is replaced by byte in row 9 column 5
  - which has value {2A}
- S-box constructed using defined transformation of values in GF(2^8)
- designed to be resistant to all known attacks

Byte Substitution

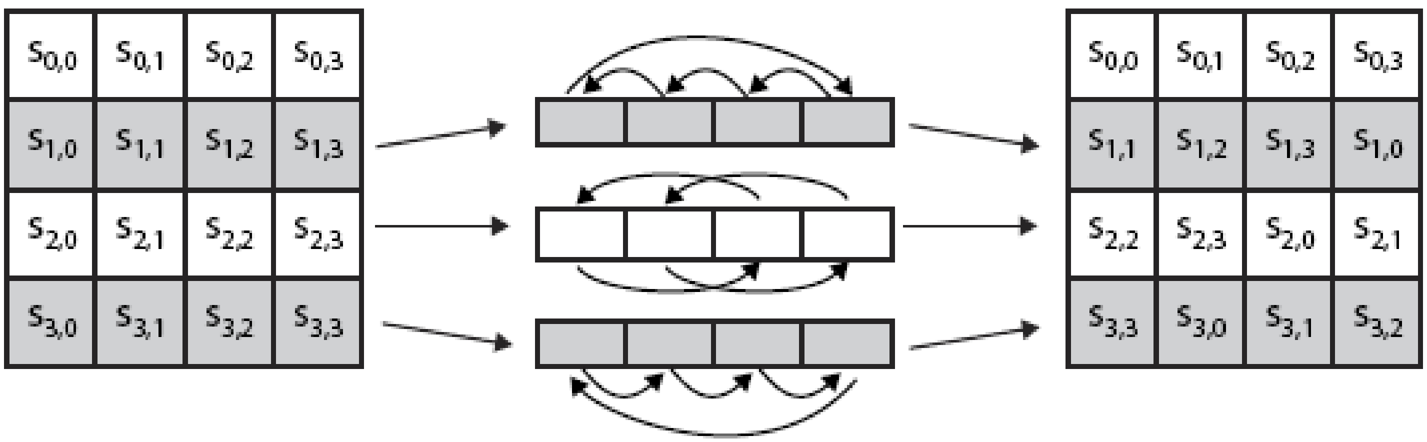
lsb / msb	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
10	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
11	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
12	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
13	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
14	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
15	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16



Shift Rows

- a circular byte shift in each each
  - 1st row is unchanged
  - 2nd row does 1 byte circular shift to left
  - 3rd row does 2 byte circular shift to left
  - 4th row does 3 byte circular shift to left
- decrypt inverts using shifts to right
- since state is processed by columns, this step permutes bytes between the columns

Shift Rows



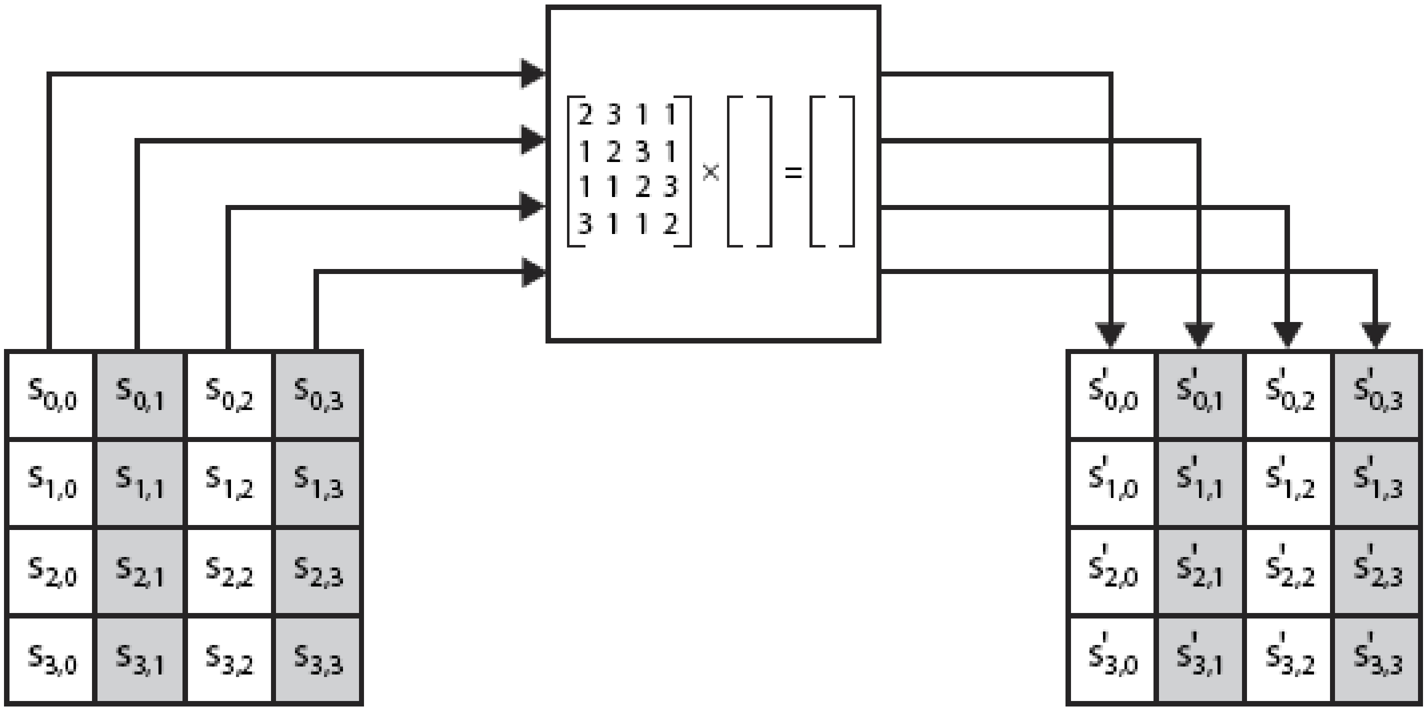
$s_{0,0}$ $s_{0,1}$ $s_{0,2}$ $s_{0,3}$
$s_{1,0}$ $s_{1,1}$ $s_{1,2}$ $s_{1,3}$
$s_{2,0}$ $s_{2,1}$ $s_{2,2}$ $s_{2,3}$
$s_{3,0}$ $s_{3,1}$ $s_{3,2}$ $s_{3,3}$
$s_{0,0}$ $s_{0,1}$ $s_{0,2}$ $s_{0,3}$
$s_{1,1}$ $s_{1,2}$ $s_{1,3}$ $s_{1,0}$
$s_{2,2}$ $s_{2,3}$ $s_{2,0}$ $s_{2,1}$
$s_{3,3}$ $s_{3,0}$ $s_{3,1}$ $s_{3,2}$

Mix Columns

- each column is processed separately
- each byte is replaced by a value dependent on all 4 bytes in the column
- effectively a matrix multiplication in $GF(2^8)$ using prime poly $m(x) = x^8+x^4+x^3+x+1$

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

Mix Columns



2 3 1 1
1 2 3 1
1 1 2 3
3 1 1 2
×
=
s0,0 s0,1 s0,2 s0,3
s1,0 s1,1 s1,2 s1,3
s2,0 s2,1 s2,2 s2,3
s3,0 s3,1 s3,2 s3,3
s'0,0 s'0,1 s'0,2 s'0,3
s'1,0 s'1,1 s'1,2 s'1,3
s'2,0 s'2,1 s'2,2 s'2,3
s'3,0 s'3,1 s'3,2 s'3,3

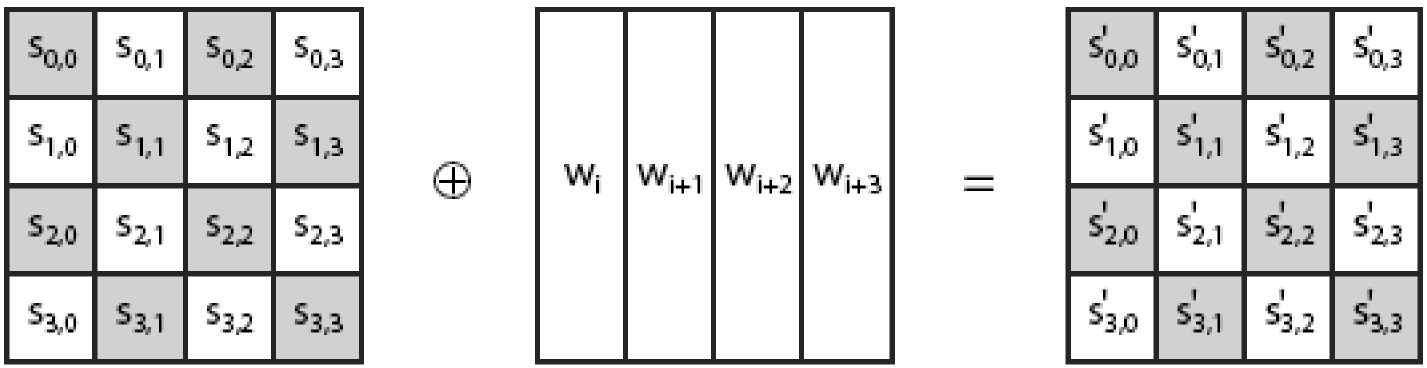
Mix Columns

- can express each col as 4 equations
  - to derive each new byte in col
- decryption requires use of inverse matrix
  - with larger coefficients, hence a little harder
- have an alternate characterisation
  - each column a 4-term polynomial
  - with coefficients in $GF(2^8)$
  - and polynomials multiplied modulo (x^4+1)

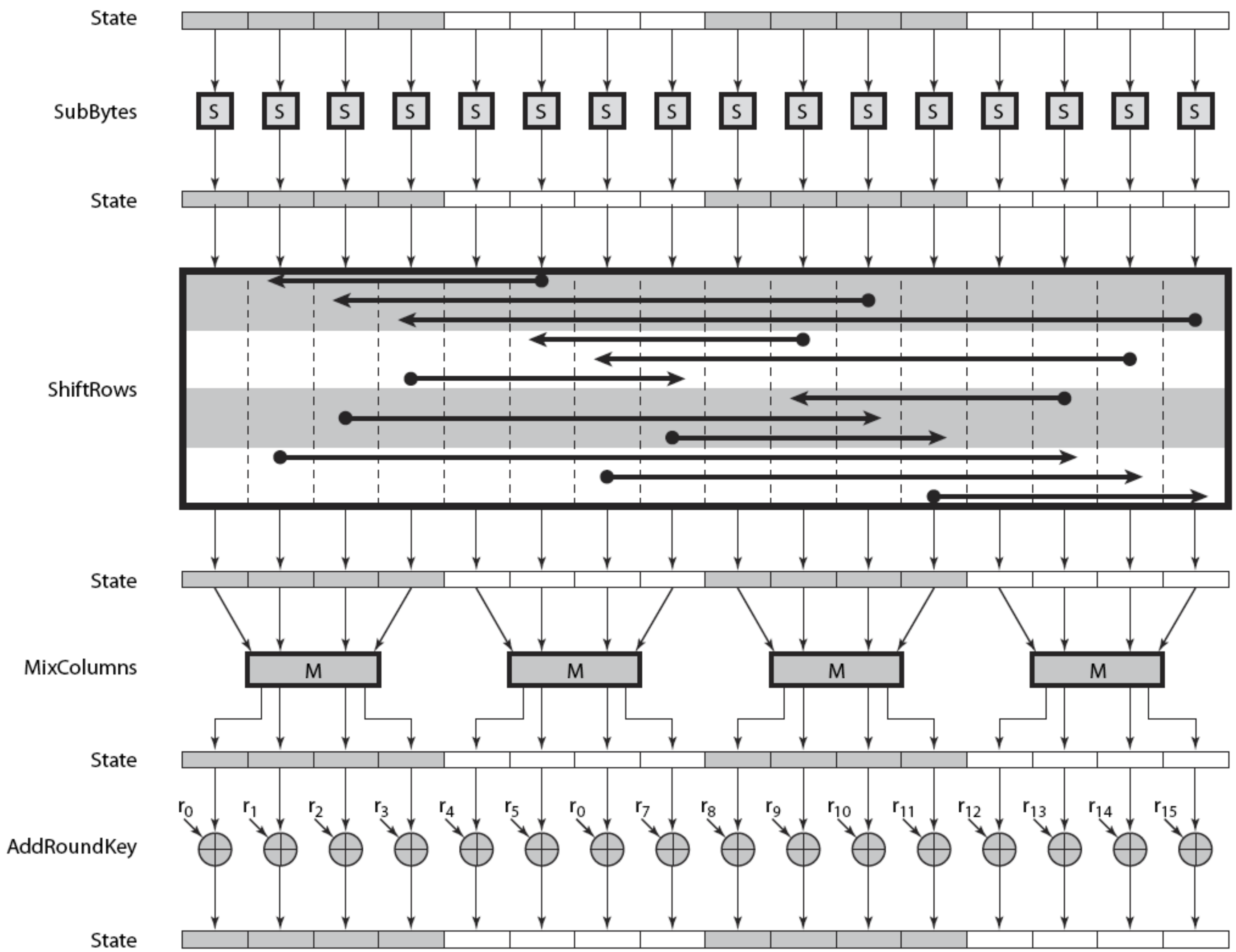
Add Round Key

- XOR state with 128-bits of the round key
- again processed by column (though effectively a series of byte operations)
- inverse for decryption identical
  - since XOR own inverse, with reversed keys
- designed to be as simple as possible
  - a form of Vernam cipher on expanded key
  - requires other stages for complexity / security

Add Round Key



AES Round

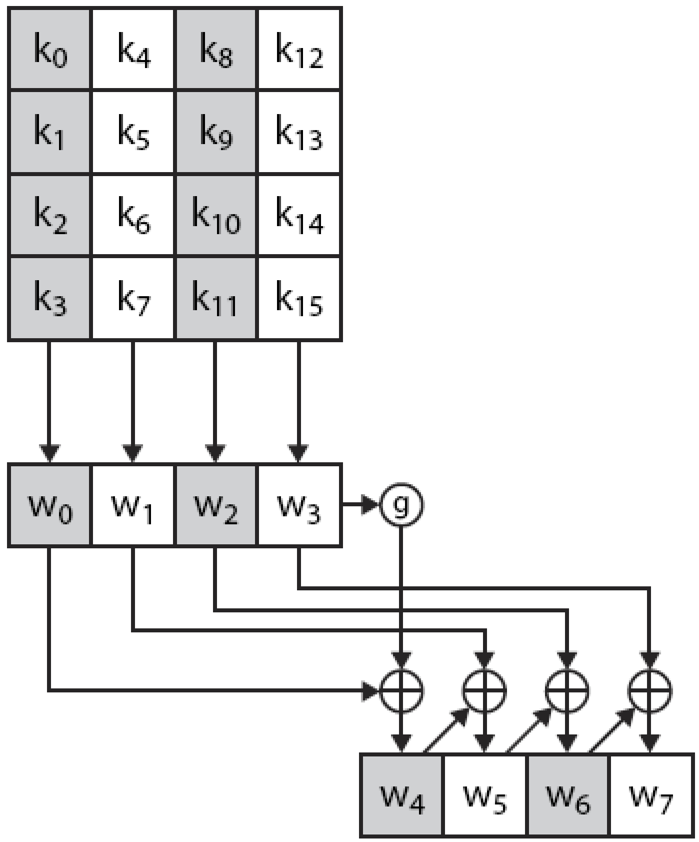


State
SubBytes
S
State
ShiftRows
State
MixColumns
M
State
AddRoundKey
r_0 r_1 r_2 r_3 r_4 r_5 r_0 r_7 r_8 r_9 r_{10} r_{11} r_{12} r_{13} r_{14} r_{15}
State

AES Key Expansion

- takes 128-bit (16-byte) key and expands into array of 44/52/60 32-bit words
- start by copying key into first 4 words
- then loop creating words that depend on values in previous & 4 places back
  - in 3 of 4 cases just XOR these together
  - 1st word in 4 has rotate + S-box + XOR round constant on previous, before XOR 4th back

AES Key Expansion



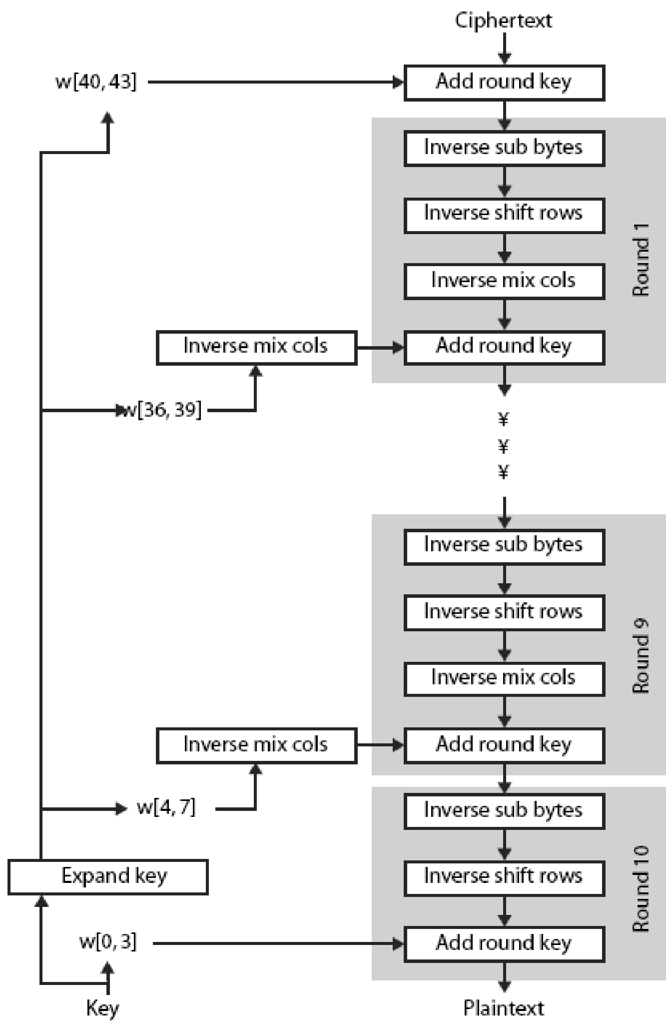
Key Expansion Rationale

- designed to resist known attacks
- design criteria included
  - knowing part key insufficient to find many more
  - invertible transformation
  - fast on wide range of CPU's
  - use round constants to break symmetry
  - diffuse key bits into round keys
  - enough non-linearity to hinder analysis
  - simplicity of description

AES Decryption

- AES decryption is not identical to encryption since steps done in reverse
- but can define an equivalent inverse cipher with steps as for encryption
  - but using inverses of each step
  - with a different key schedule
- works since result is unchanged when
  - swap byte substitution & shift rows
  - swap mix columns & add (tweaked) round key

AES Decryption



Implementation Aspects

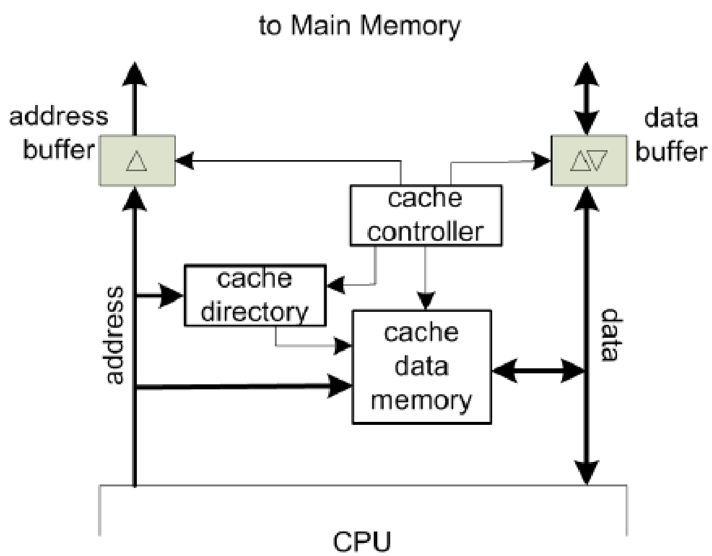
- can efficiently implement on 8-bit CPU
  - byte substitution works on bytes using a table of 256 entries
  - shift rows is simple byte shift
  - add round key works on byte XOR's
  - mix columns requires matrix multiply in $GF(2^8)$ which works on byte values, can be simplified to use table lookups & byte XOR's

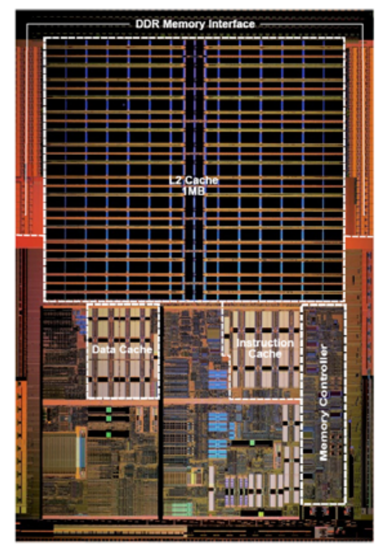
Implementation Aspects

- can efficiently implement on 32-bit CPU
  - redefine steps to use 32-bit words
  - can precompute 4 tables of 256-words
  - then each column in each round can be computed using 4 table lookups + 4 XORs
  - at a cost of 4Kb to store tables
- designers believe this very efficient implementation was a key factor in its selection as the AES cipher

Security of AES

- No crypt-analytical weaknesses for 10 round version
  - Shorter round versions of AES are provably less secure.
- Optimized implementation easiness produced several
  - Fundamental implementation security issues.
  - Difficulties for fast and secure implementations in hw and sw.





Homework suggestion (for next lecture)

- Please read the following papers:
  - D. J. Bernstein, "Cache-timing attacks on AES," http://cr.yp.to/antiforgery/cachetiming-20050414.pdf
  - O. Acıiçmez, J.-P Seifert, and C. Koç, "Micro-Architectural Cryptanalysis," ***IEEE Trans. Security and Privacy,*** vol. 5, no. 4, Jul.-Aug. 2007, pp. 62-64.

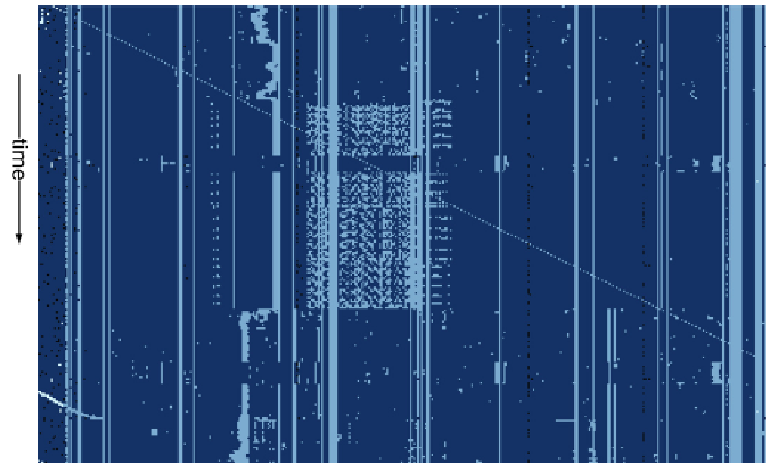


Figure 4: Evolution of the cache *versus* time. Each horizontal line represents the state of the cache lines (represented by a point) at a given time. Different AES encryption are clearly visible in the center. The brighter a point, the longer the time to access its corresponding cache line.

Private-Key Cryptography

- traditional **private/secret/single key** cryptography uses **one** key
- shared by both sender and receiver
- if this key is disclosed communications are compromised
- also is **symmetric**, parties are equal
- hence does not protect sender from receiver forging a message & claiming is sent by sender

Public-Key Cryptography

- probably most significant advance in the 3000 year history of cryptography
- uses **two** keys –
  - a public & a private key
- **asymmetric** since parties are **not** equal
- uses clever application of number theoretic concepts to function
- complements **rather than** replaces private key crypto

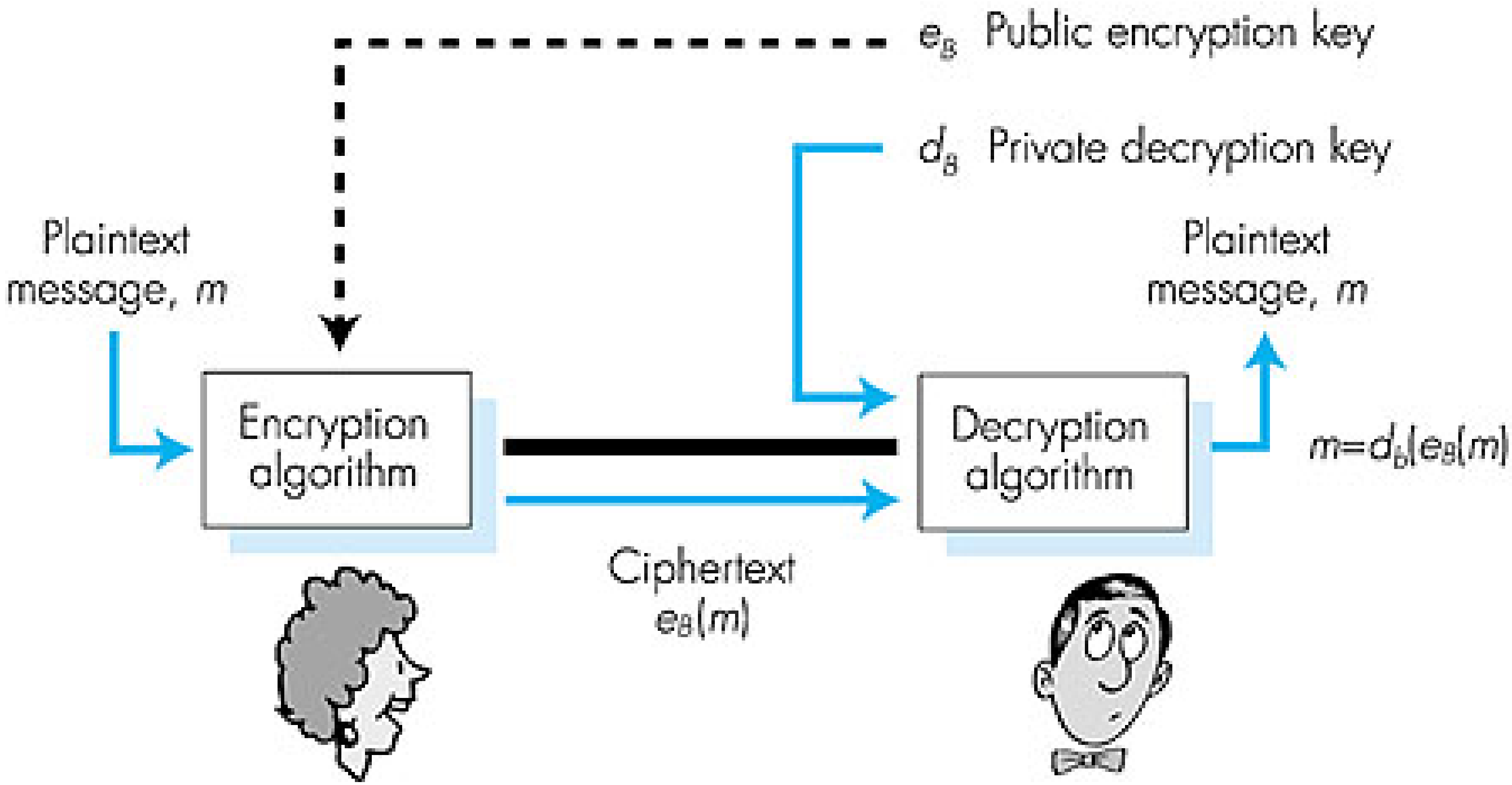
Why Public-Key Cryptography?

- developed to address two key issues:
  - **key distribution** – how to have secure communications in general without having to trust a KDC with your key
  - **digital signatures** – how to verify a message comes intact from the claimed sender
- public invention due to Whitfield Diffie & Martin Hellman at Stanford Uni in 1976
  - known earlier in classified community

Public-Key Cryptography

- **public-key/two-key/asymmetric** cryptography involves the use of **two** keys:
  - a **public-key**, which may be known by anybody, and can be used to **encrypt messages**, and **verify signatures**
  - a **private-key**, known only to the recipient, used to **decrypt messages**, and **sign** (create) **signatures**
- is **asymmetric** because
  - those who encrypt messages or verify signatures **cannot** decrypt messages or create signatures

Public key cryptography



Public-Key Characteristics

- Public-Key algorithms rely on two keys where:
  - it is computationally infeasible to find decryption key knowing only algorithm & encryption key
  - it is computationally easy to en/decrypt messages when the relevant (en/decrypt) key is known
  - either of the two related keys can be used for encryption, with the other used for decryption (for some algorithms)

Public-Key Applications

- can classify uses into 3 categories:
  - **encryption/decryption** (provide secrecy)
  - **digital signatures** (provide authentication)
  - **key exchange** (of session keys)

- some algorithms are suitable for all uses, others are specific to one

Security of Public Key Schemes

- like private key schemes brute force **exhaustive search** attack is always theoretically possible
- but keys used are too large (>512bits)
- security relies on a **large enough** difference in difficulty between **easy** (en/decrypt) and **hard** (crypt-analyse) problems
- more generally the **hard** problem is known, but is made hard enough to be impractical to break
- requires the use of **very large numbers**
- hence is **slow** compared to private key schemes

Public Key Cryptography

Symmetric key crypto

- Requires sender, receiver to know shared secret key
- Q: how to agree on key in first place (particularly if never "met")?
- Q: what if key is stolen?
- Q: what if you run out of keys?
- Q: what if A doesn't know she wants to talk to B?

Public key cryptography

- Radically different approach [Diffie-Hellman76, RSA78]
- Sender, receiver do *not* share secret key
- Encryption key *public* (known to *all*)
- Decryption key private (known only to receiver)
- Allows parties to communicate without prearrangement

Prime Numbers

- prime numbers only have divisors of 1 and self
  - they cannot be written as a product of other numbers
  - note: 1 is prime, but is generally not of interest
- eg. 2,3,5,7 are prime, 4,6,8,9,10 are not
- prime numbers are central to number theory
- list of prime number less than 200 is:

  2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167 173 179 181 191 193 197 199

Relatively Prime Numbers & GCD

- two numbers `a,b` are **relatively prime** if have **no common divisors** apart from 1
  - eg. 8 & 15 are relatively prime since factors of 8 are 1,2,4,8 and of 15 are 1,3,5,15 and 1 is the only common factor
- conversely can determine the greatest common divisor by comparing their prime factorizations and using least powers
  - eg. $300=2^1 \times 3^1 \times 5^2$ $18=2^1 \times 3^2$ hence $GCD(18,300)=2^1 \times 3^1 \times 5^0=6$

Fermat's Theorem

- $a^{p-1} = 1 \pmod{p}$
  - where p is prime and $\gcd(a,p)=1$
- also known as Fermat's Little Theorem
- also $a^p = p \pmod{p}$
- useful in public key and primality testing

Euler Totient Function $\varnothing$(n)

- when doing arithmetic modulo n
- **complete set of residues** is: `0..n-1`
- **reduced set of residues** is those numbers (residues) which are relatively prime to n
  - eg for n=10,
  - complete set of residues is {0,1,2,3,4,5,6,7,8,9}
  - reduced set of residues is {1,3,7,9}
- number of elements in reduced set of residues is called the **Euler Totient Function ø(n)**

Euler Totient Function $\varnothing(n)$

- to compute ø(n) need to count number of residues to be excluded
- in general need prime factorization, but
  - for p (p prime) $\varnothing(p) = p-1$
  - for p.q (p,q prime) $\varnothing(pq) = (p-1)x(q-1)$
- eg.

  $\varnothing(37) = 36$

  $\varnothing(21) = (3-1)x(7-1) = 2x6 = 12$

Euler's Theorem

- a generalisation of Fermat's Theorem
- $a^{\varnothing(n)} = 1 \pmod{n}$
  - for any a,n where gcd(a,n)=1
- eg.

  $a=3; n=10;\ \varnothing(10)=4;$

  hence $3^4 = 81 = 1 \bmod 10$

  $a=2; n=11;\ \varnothing(11)=10;$

  hence $2^{10} = 1024 = 1 \bmod 11$

Primitive Roots

- from Euler's theorem have $a^{\varnothing(n)} \bmod n=1$
- consider $a^m=1 \pmod n$, $GCD(a,n)=1$
  - must exist for $m = \varnothing(n)$ but may be smaller
  - once powers reach m, cycle will repeat
- if smallest is $m = \varnothing(n)$ then a is called a **primitive root** or **generating element**
- if p is prime, then successive powers of a "generate" the group $\bmod p$
- these are useful but relatively hard to find

Discrete Logarithms

- the inverse problem to exponentiation is to find the **discrete logarithm** of a number modulo p
- that is to find x such that $y = g^x \pmod{p}$
- this is written as $x = \log_g y \pmod{p}$
- if g is a primitive root then it always exists, otherwise it may not, eg.
  - $x = \log_3 4 \bmod 13$ has no answer
  - $x = \log_2 3 \bmod 13 = 4$ by trying successive powers
- whilst exponentiation is relatively easy, finding discrete logarithms is generally a **hard** problem

Public-Key distribution of Secret Keys

- use previous methods to obtain public-key
- can use for secrecy or authentication
- but public-key algorithms are slow
- so usually want to use private-key encryption to protect message contents
- hence need a session key
- have several alternatives for negotiating a suitable session

Diffie-Hellman Key Exchange

- first public-key type scheme proposed
- by Diffie & Hellman in 1976 along with the exposition of public key concepts
  - note: now known that Williamson (UK CESG) secretly proposed the concept in 1970
- is a practical method for public exchange of a secret key
- used in a number of commercial products

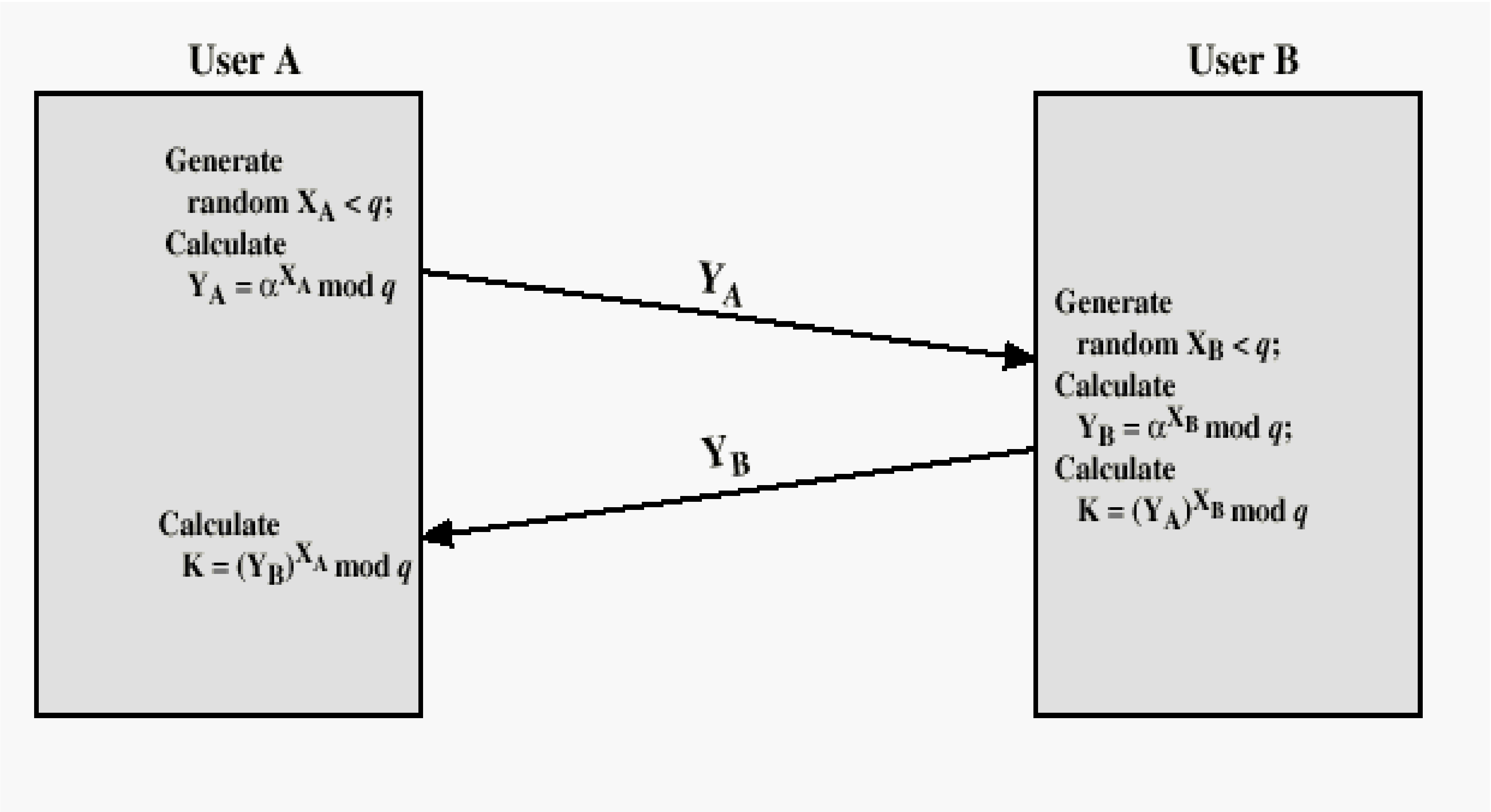
Diffie-Hellman Key Exchange

- a public-key distribution scheme
  - cannot be used to exchange an arbitrary message
  - rather it can establish a common key
  - known only to the two participants
- value of key depends on the participants (and their private and public key information)
- based on exponentiation in a finite (Galois) field (modulo a prime or a polynomial) – seems easy at first sight
- security relies on the difficulty of computing discrete logarithms (similar to factoring) – hard

Diffie-Hellman Setup

- all users agree on global parameters:
  - large prime integer or polynomial q
  - a being a primitive root mod q
- each user (eg. A) generates their key
  - chooses a secret key (number): $x_A < q$
  - compute their **public key**: $y_A = a^{x_A} \bmod q$
- each user makes public that key y_A

Diffie-Hellman Key Echange



Diffie-Hellman Key Exchange

- shared session key for users A & B is K_{AB}:

  $K_{AB} = a^{x_A . x_B} \bmod q$

  $= y_A^{x_B} \bmod q$ (which **B** can compute)

  $= y_B^{x_A} \bmod q$ (which **A** can compute)

- K_{AB} is used as session key in private-key encryption scheme between Alice and Bob

- if Alice and Bob subsequently communicate, they will have the **same** key as before, unless they choose new public-keys

- attacker needs an x, thus must solve discrete log, logarithm modulo q, i.e,., compute x_A from $y_A = a^{x_A}$

Diffie-Hellman Example

- users Alice & Bob who wish to swap keys:
- agree on prime $q=353$ and $a=3$
- select random secret keys:
  - A chooses $x_A=97$, B chooses $x_B=233$
- compute respective public keys:
  - $y_A=3^{97} \bmod 353 = 40$ (Alice)
  - $y_B=3^{233} \bmod 353 = 248$ (Bob)
- compute shared session key as:
  - $K_{AB}= y_B^{x_A} \bmod 353 = 248^{97} = 160$ (Alice)
  - $K_{AB}= y_A^{x_B} \bmod 353 = 40^{233} = 160$ (Bob)

Key Exchange Protocols

- users could create random private/public D-H keys each time they communicate
- users could create a known private/public D-H key and publish in a directory, then consulted and used to securely communicate with them
- both of these are vulnerable to a meet-in-the-Middle Attack
- authentication of the keys is needed
  - Next lectures more on this!

Summary

- Have considered:
  - Details of Rijndael – the AES cipher
  - Principle of Public Key Cryptography
  - Number Theory basics
  - Diffie-Hellmann Key exchange