

Time-series and scaling analysis with R

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Generate heavy-tailed distribution

- `library(VGAM)`
- Generating Pareto distribution:
 - `vector.pareto1 <- rpareto(n=100000,location=10,shape=1)`
 - `vector.pareto1.5 <- rpareto(n=100000,location=10,shape=1.5)`
 - `vector.pareto2 <- rpareto(n=100000,location=10,shape=2)`
- Plot CCDF on a log-log scale:
 - ```
ccdf <- function(myList,density=FALSE)
{
 # generates the CCDF of a list or vector
 freqs = table(myList)
 X = rev(as.numeric(names(freqs)))
 Y = cumsum(rev(as.list(freqs)));
 data.frame(x=X,count=Y)
}
```
  - `plot(ccdf(vector.pareto1),log="xy")`

# Time-series aggregation

- `library(tseries)`
- Computing m-aggregated time series:
  - `vector.ts <- ts(vector,frequency=m)`
  - `vector.m-agg <- aggregate(vector.ts,FUN=mean)`
- Plot aggregated time series:
  - `plot(vector.m-agg)`
- Compare variances of aggregated time-series:
  - poisson decreases proportionately with m
  - heavy-tailed very slowly

# Auto-correlation

- `library(stats)`
- Auto-correlation function: `acf()`
  - Poisson process is uncorrelated: 1 at lag 0, 0 otherwise
  - Pareto time-series is also uncorrelated: 1 at lag 0, 0 otherwise
  - Real traffic is correlated: `acf(vector,log="y")` shows slow decay

# Wavelets

- library(waveslim)
- Performing wavelet analysis:
  - `vector.res <- dwt(vector [1:2**16],wf="d16",n.levels=10,boundary="periodic")`
  - Length of analyzed time-series (L) must be a power of 2
  - Several wavelet functions available (wf)
  - Number of levels to be analyzed:  $2^n \leq L$

# Wavelets

- Selecting wavelet coefficients at scale  $i$ 
  - `vector.res$di`
- Computing  $\log_2$  of their variance:
  - `log2(var((res.vector$di)))`
- Keeping energy of coefficients at each scale:
  - `coefvar <- mat.or.vec(9,1)`
  - `coefvar[i] <- log2(var((res.vector$di)))`
- Plot the log of the energy of the wavelet coefficients
  - Poisson process shows no scaling
  - Real traffic does