Δ-Reliable Broadcast:
A Probabilistic Measure of Broadcast Reliability

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Outline

1. The problem: scalable “reliable” broadcast
2. Reliable Broadcast specification [HT94].
3. $\Delta$-Reliable Broadcast.
4. Reliability distribution function.
5. Examples: reliability analysis of Bimodal Multicast [BHO$^+$99] and IP Multicast [DC90].
Broadcast protocols

• Best-effort: Multicast Usenet (MUSE), IP Multicast, Reliable Multicast Transfer Protocol (RMTP), etc.

• Probabilistic: Bimodal Multicast, lpbcast, etc.

What is the problem?
Traditional specification: Reliable Broadcast[HT94]

Integrity. For any message $m$, every process delivers $m$ at most once, and only if $m$ was previously broadcast by $\text{sender}(m)$.

Validity. If a correct process $p$ broadcasts a message $m$, then $p$ eventually delivers $m$.

Agreement. If a correct process delivers a message $m$, then every correct process eventually delivers $m$. 
Informal specification: Atomicity [BHO+99]

A broadcast protocol provides a bimodal delivery guarantee if there is

- a high probability that a broadcast message will reach almost all processes,
- a low probability that a broadcast message will reach just a very small set of processes, and
- a vanishingly small probability that a broadcast message will reach some intermediate number of processes.
Bridging the gap: $\Delta$-Reliable Broadcast

Let $\Delta = (\psi, \rho) \in [0, 1] \times [0, 1]$. A broadcast protocol is $\Delta$-Reliable iff the following properties are simultaneously satisfied with probability $\psi$:

**Integrity.** For any message $m$, every process delivers $m$ at most once, and only if $m$ was previously broadcast by $sender(m)$.

**Validity.** If a correct process $p$ broadcasts a message $m$ then $p$ eventually delivers $m$.

**$\Delta$-Agreement.** If a correct process delivers a message $m$, then eventually at least a fraction $\rho$ of correct processes deliver $m$. 
\( \Delta \)-Reliable Broadcast: \( \rho \) and \( \psi \)

\( \Delta = (\psi, \rho) \) is a “reliability measure” of a given protocol.

**Reliability degree** \( \rho \): the fraction of correct processes that eventually deliver a broadcast message.

**Reliability probability** \( \psi \): - the probability that “enough” (correct) processes deliver a broadcast message and no fake or duplicate messages are delivered.
Reliability distribution function

Let $\mathcal{E}$ be an *environment space* and $B$ be a broadcast protocol.

A function $\psi_B : [0, 1] \times \mathcal{E} \mapsto [0, 1]$ is the *reliability distribution function* of $B$ iff

$$\forall \rho \in [0, 1] \ \forall \mathcal{E} \in \mathcal{E}: \ B \text{ is } \Delta \text{-Reliable with } \Delta = (\psi_B(\rho, \mathcal{E}), \rho).$$

$B_1$ is more reliable than $B_2$ in $\mathcal{E}$. 
Reliability distribution function: examples

• *Dreamcast* (Reliable Broadcast [HT94]) in a given $\mathcal{E} \in \mathcal{E}$:

$$\forall \rho \in [0, 1] : \psi(\rho, \mathcal{E}) = 1.$$ 

• *Spellcast* (does nothing):

$$\forall \rho \in ]0, 1], \forall \mathcal{E} \in \mathcal{E} : \psi(\rho, \mathcal{E}) = 0.$$
Atomicity

Atomicity predicate of Bimodal Multicast: given a protocol $B$, $\sigma \in [0, 0.5]$ and an environment $\mathcal{E} \in \mathbb{E}$, a broadcast message reaches more than a fraction $\sigma$, but less than a fraction $1 - \sigma$ of correct processes with probability:

$$P(\sigma \leq \rho < 1 - \sigma) = \psi_B(\sigma, \mathcal{E}) - \psi_B(1 - \sigma, \mathcal{E})$$
Bimodal Multicast \([\text{BHO}^+99]\)

Environment:
- \(n\) processes
- Fanout \(\beta\)
- Message loss probability \(\varepsilon\)
- Process crash probability \(\tau\)
- Number of gossip rounds \(T\)

**deliver_and_gossip\((m, \text{round})\)** \{ \text{* Auxiliary function *} \}
- if received\(_{\text{already}}\)\((m)\) then return \(\text{bmdeliver}(m)\)
- if \(\text{round}=T\) then return
- choose \(S \subseteq \Pi\), such that \(|S| = n\beta\)
- for each \(p\) in \(S\) send to \(p\) \(\text{gossip}(m,\text{round}+1)\)

On \(\text{bmcast}(m)\):
\[
\text{deliver_and_gossip}(m,0)
\]

On receive \(\text{gossip}(m,\text{round})\):
\[
\text{deliver_and_gossip}(m,\text{round})
\]
Environmental Assumptions:

- A $k$-ary spanning tree of depth $d$
- $k^d$ processes
- Message loss probability $\epsilon$
- Process crash probability $\tau$
- Router crash probability $\gamma$
Reliability distribution functions

![Graph showing reliability distribution functions for Bimodal Multicast and IP Multicast.](image-url)
Average reliability degrees

![Graph showing expected reliability degrees for Bimodal Multicast and IP Multicast. The graph plots expected reliability degree on the y-axis against n on the x-axis. The graph indicates that Bimodal Multicast has a higher reliability degree compared to IP Multicast.]
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References


