# Walking through the Cloud: Routing of Virtualized Network Functions

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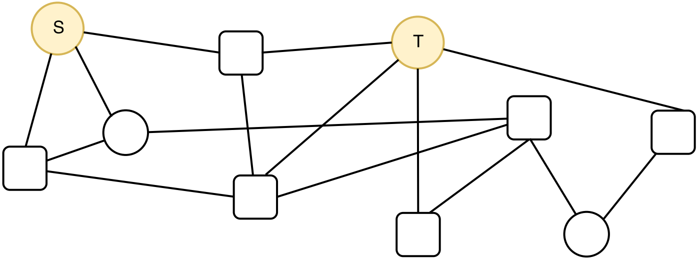
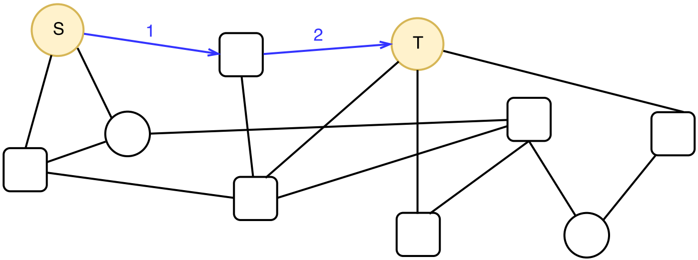
ALGOCLOUD, 2017

# A Routing Problem



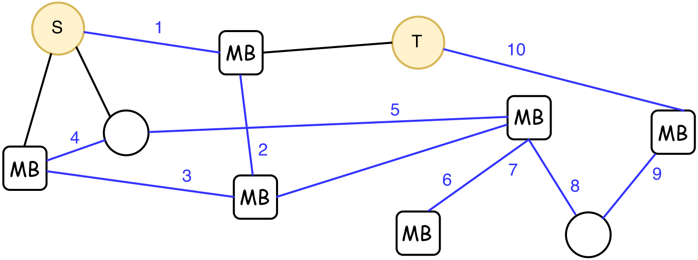
# A Routing Problem
## the usual routing



# A Routing Problem

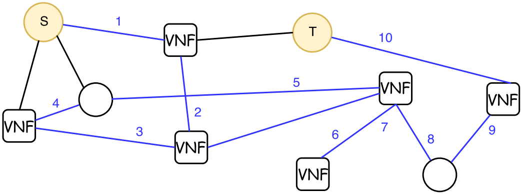
new: middleboxes

- Middlebox: Firewall, NAT, proxies, DPI etc.



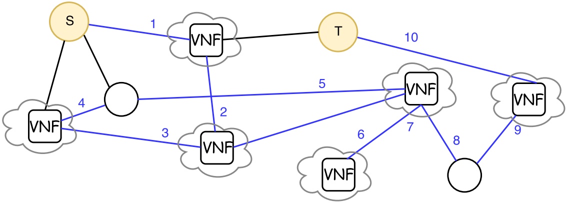
# A Routing Problem
## moving to clouds

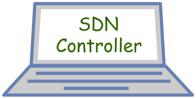
- VNFs brings flexibility, are cheaper



# A Routing Problem
## moving to clouds

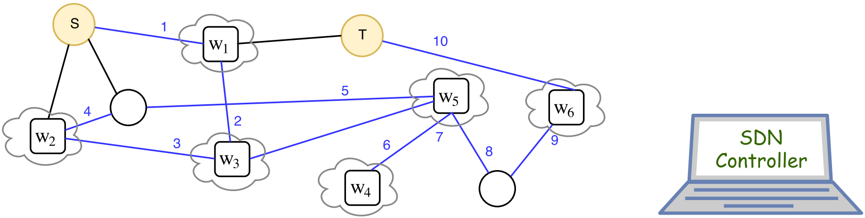
- VNFs brings flexibility, are cheaper
- A lot of them, in clouds



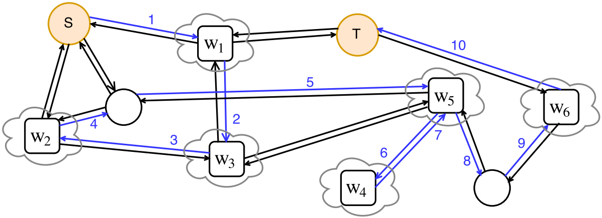
# A Routing Problem
## the waypoints

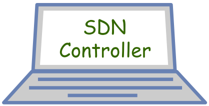
- The task: find the shortest $S$–$T$ walk through *waypoints*
- Capacities must be respected



# A Routing Problem
## the network

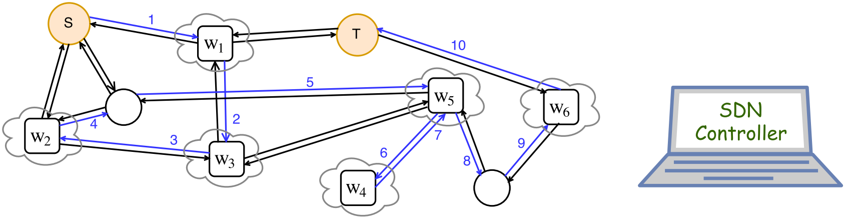
- Real Networks Are Bidirected



# A Routing Problem
## in two flavors

- Real Networks Are Bidirected
- Two Flavors: Ordered vs Unordered



# Outline

# Model

- Bidirected graph $G(V, E)$: $\forall (x, y) \in E \implies (y, x) \in E$
- $n$ nodes, $k$ of which are waypoints
- Arbitrary capacities, unit demand for $(S, T)$

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- A feasible walk respects link capacities

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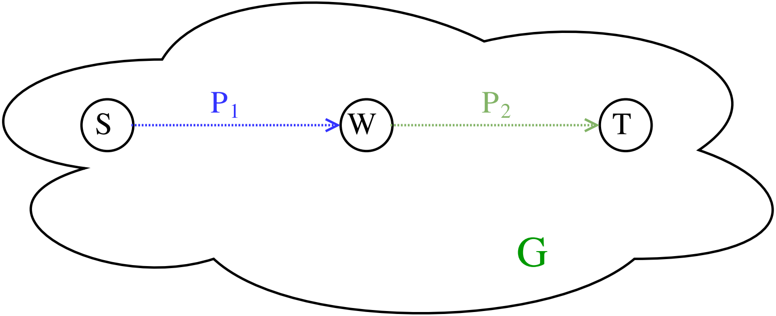
- Ordered and Unordered

# Outline

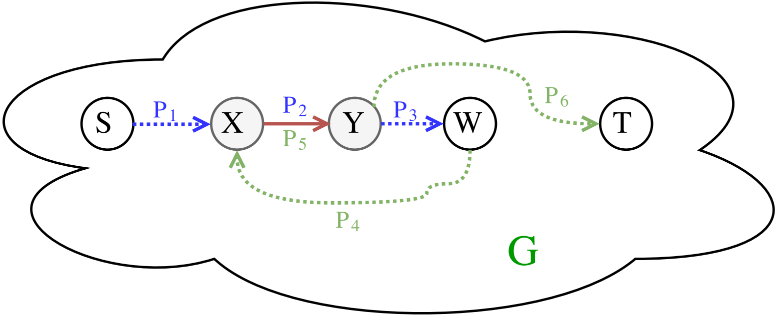
# One waypoint: greedy is optimal

Two shortest paths?



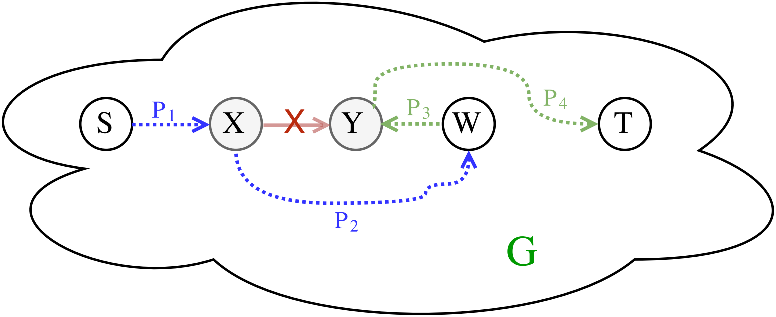
# One waypoint: greedy is optimal

Assume both shortest paths chose $(X, Y)$



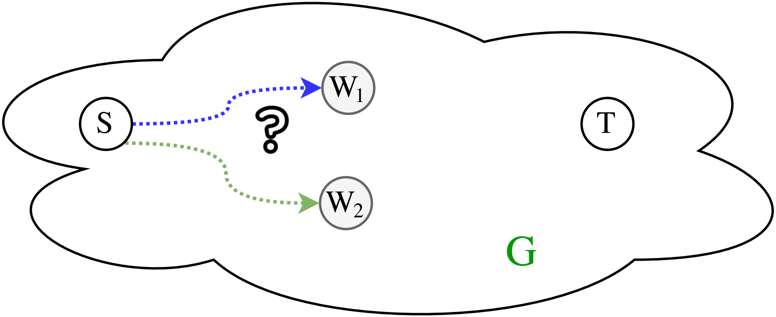
# One waypoint: greedy is optimal

$P_1P_2P_3P_4$ is shorter $\implies$ Contradiction!



# More waypoints

✓ The optimal order + shortest paths $\Longrightarrow$ it works!



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- ✓ The optimal order + shortest paths $\implies$ it works!
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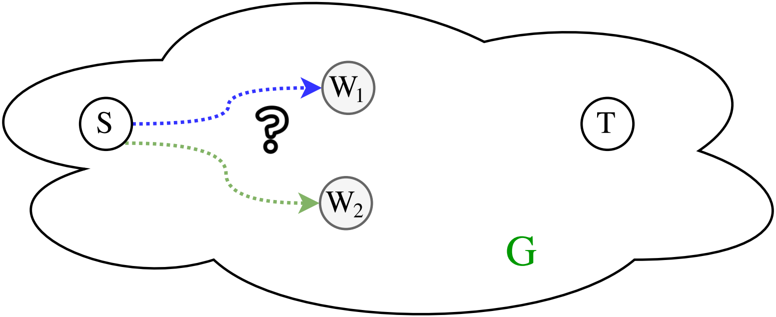

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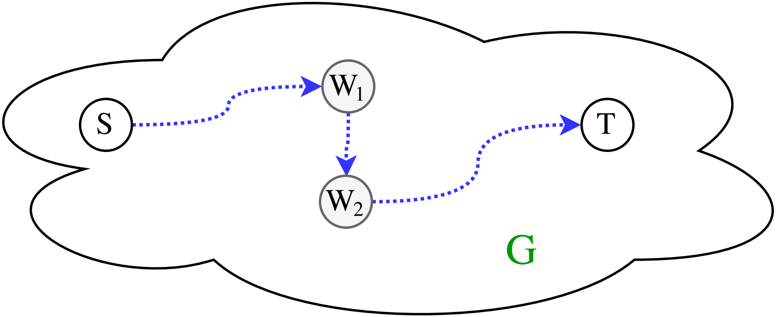

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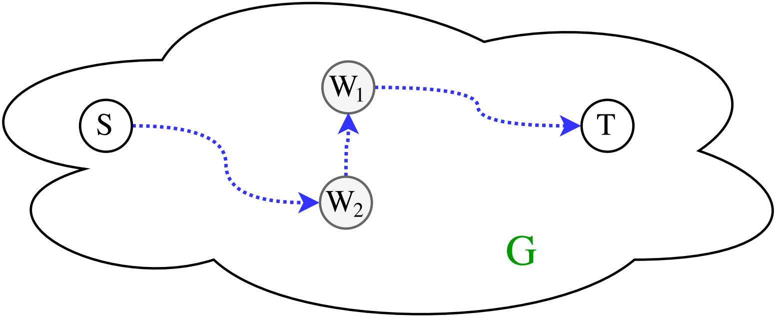
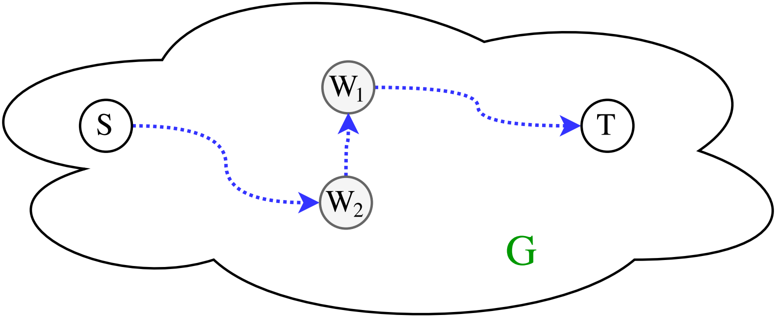


# More waypoints

- ✓ The optimal order + shortest paths $\implies$ it works!
- ✓ Try all permutations: $SW_1W_2T$ or $SW_2W_1T$?
- ✓ Polynomial time for $k = \mathcal{O}\left(\frac{\log n}{\log \log n}\right)$



# Outline

# Hardness

- Feasibility via spanning tree $\implies$ always feasible

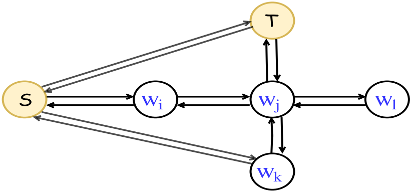



Figure: Bidirected

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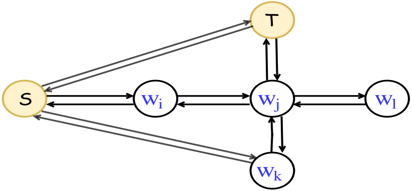


Figure: Bidirected

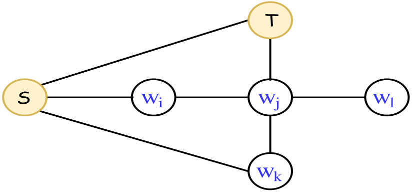


Figure: Undirected

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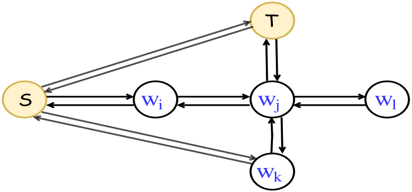


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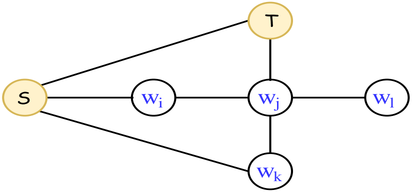


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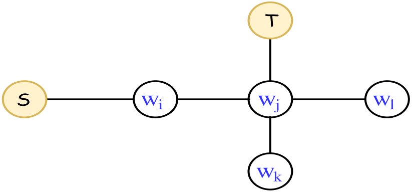


Figure: Spanning Tree

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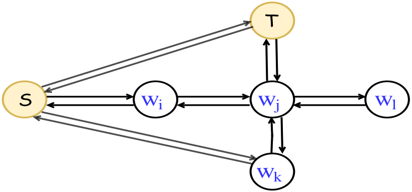


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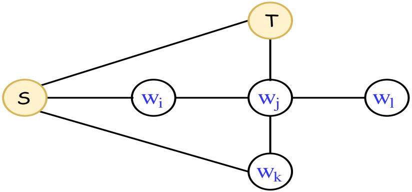


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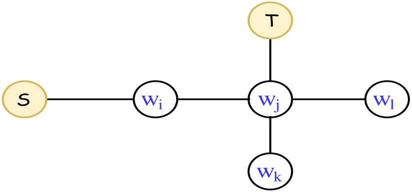


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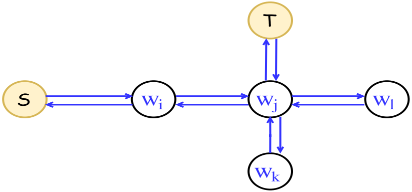


Figure: Bidirected again

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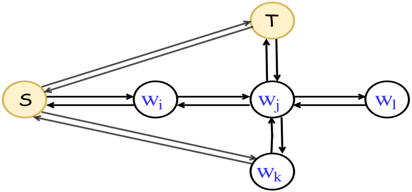


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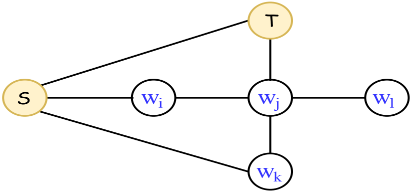


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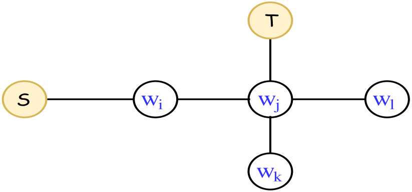


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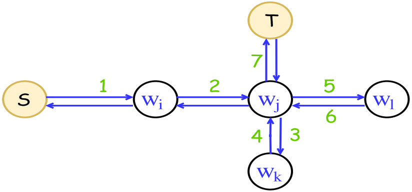


Figure: $S$-$T$ tour

# Hardness

- Feasibility via spanning tree $\Longrightarrow$ always feasible
- Approximation via metric TSP $\Longrightarrow$ L: ≈1.008[1], U: ≈1.53[2]

[1] Karpinski et al. J. Comput. Syst. Sci., 2015
[2] Andr as Seb o and Anke van Zuylen, FOCS 2016

# Hardness

- Feasibility via spanning tree $\implies$ always feasible
- Approximation via metric TSP $\implies$ L: $\approx 1.008$[1], U: $\approx 1.53$[2]
- FPT via subset TSP $\implies 2^k \cdot n^{\mathcal{O}(1)}$ (Klein and Marx, 2014)

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# Outline

# Ordered waypoint routing
## The problem

- A permutation is given, e.g. $w_1 w_2 \ldots w_k$
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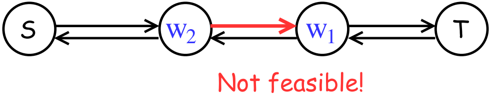

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- Related to **E**dge **D**isjoint **P**ath **P**roblem

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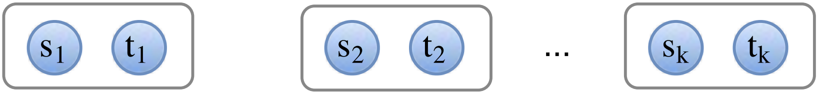
- NP-Hardness and feasibility via EDPP

# Edge Disjoint Path Problem
EDPP

**Edge Disjoint Path Problem $\in$ NP-Complete**

Find a set of pairwise edge-disjoint paths connecting every pair $(s_i, t_i), i = 1 \ldots k$

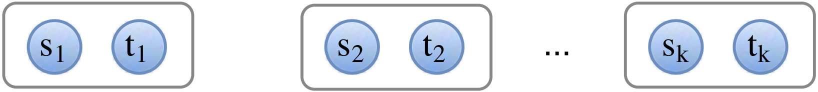


# Reduction
from EDPP

## Building the OWRP instance

The waypoints:

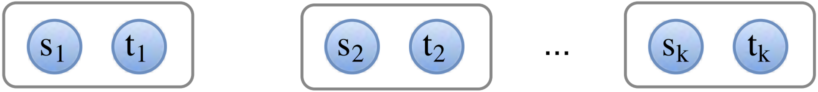
$s_1, t_1 \ldots s_i, t_i, s_{i+1}, t_{i+1} \ldots s_k, t_k$



# Reduction
from EDPP

## Building the OWRP instance

The waypoints:
$S = s_1, t_1 \dots s_i, t_i, s_{i+1}, t_{i+1} \dots s_k, t_k = T$



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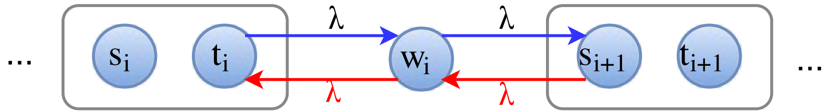

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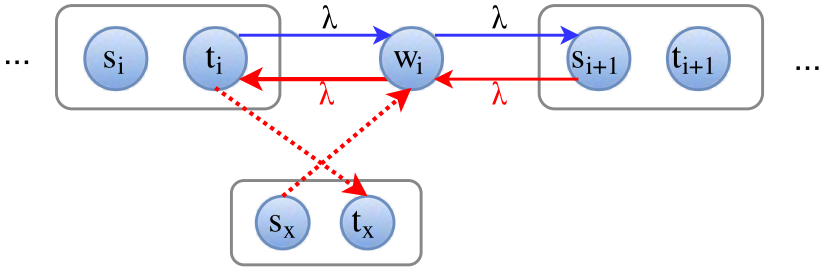

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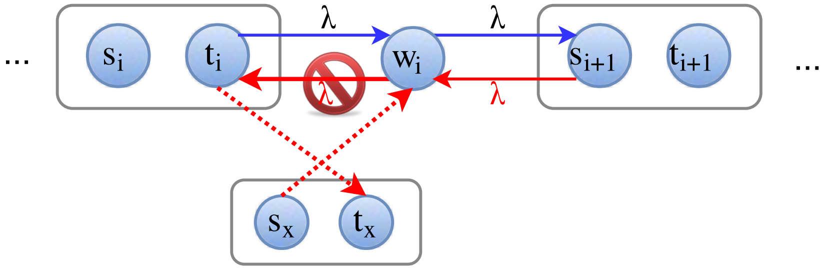

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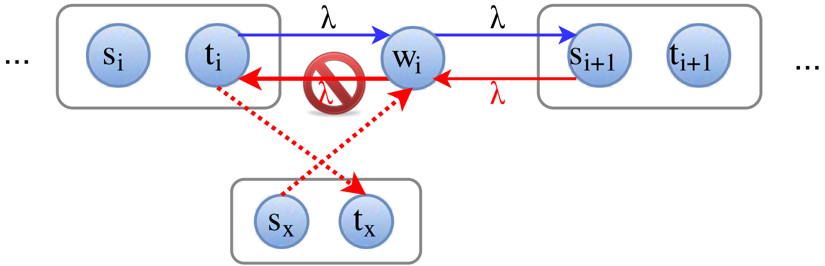


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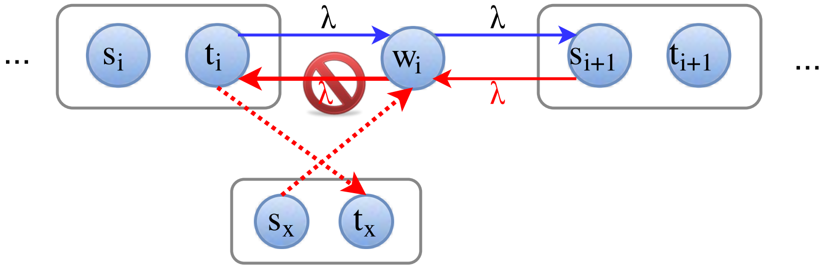


- Set $\lambda$ large enough
- OWRP chooses a backward edge $\iff$ EDPP is not feasible

# Ordered waypoint routing
## Results

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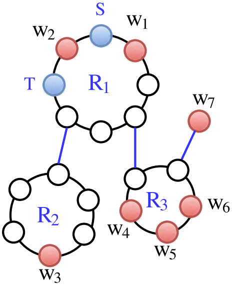
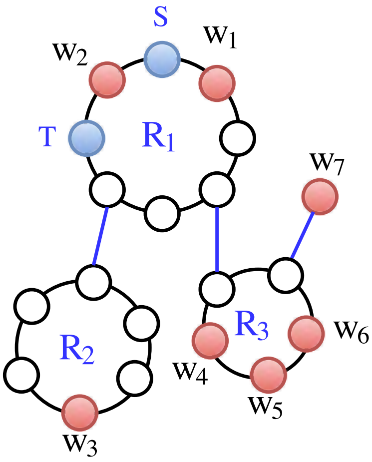
- *Cactus graph*: tree of rings



# Ordered waypoint routing

## Cactus: a tree of rings




Step 1: solve the tree contraction given

$(S = R_1), R_2, R_3, W_7, (R_1 = T)$

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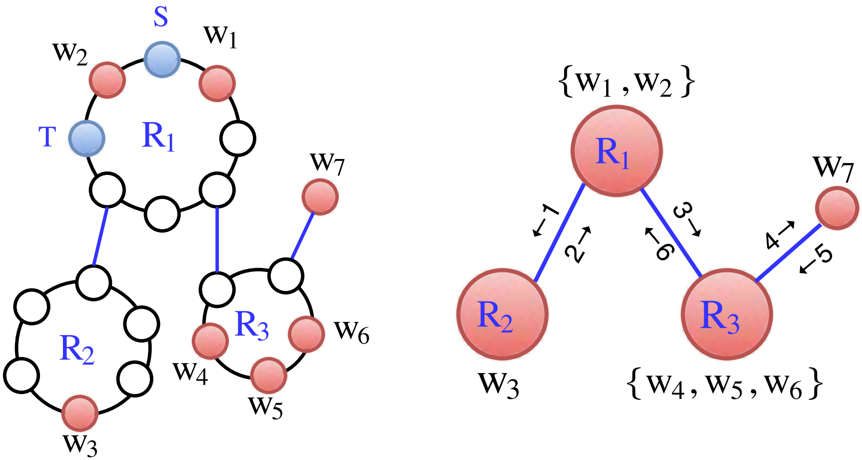


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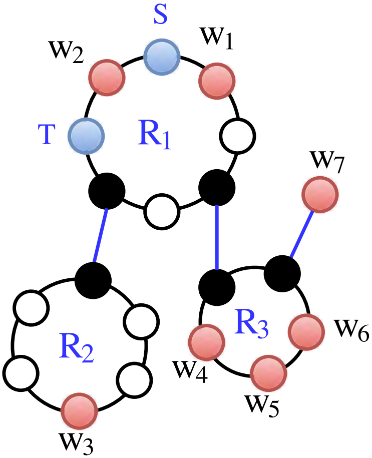


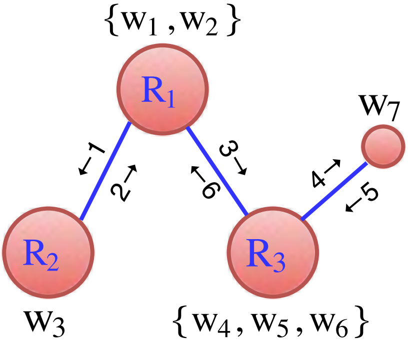


- Step 2: mark the port nodes (shown in back) as new waypoints

# Ordered waypoint routing

## Cactus: a tree of rings

- Step 2: mark the port nodes (shown in back) as new waypoints
- Step 3: solve OWRP on each ring separately

# Ordered waypoint routing

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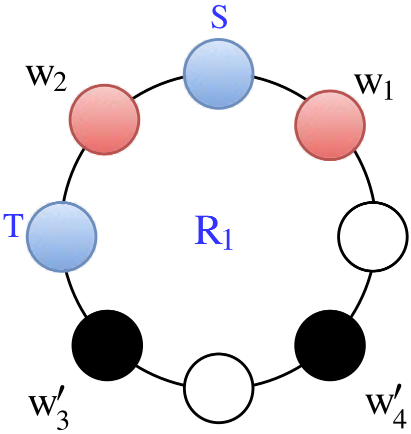

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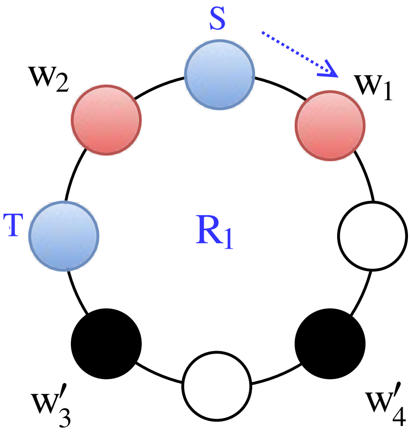

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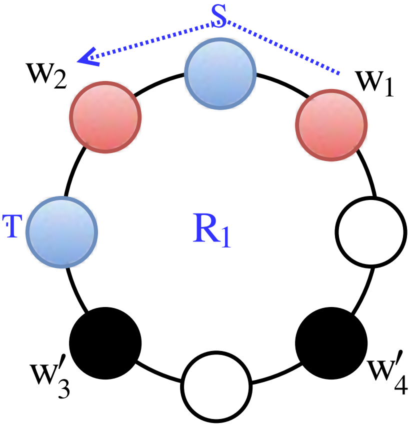

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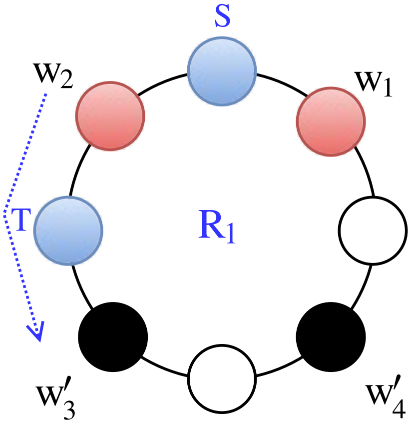
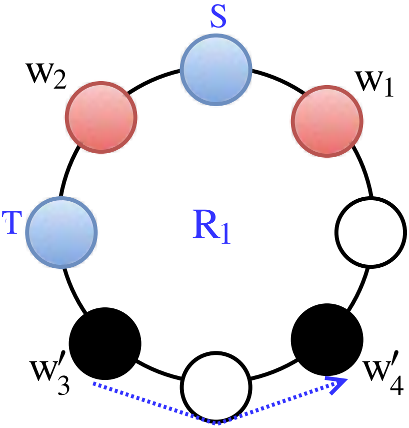


# Ordered waypoint routing

## Cactus: a tree of rings



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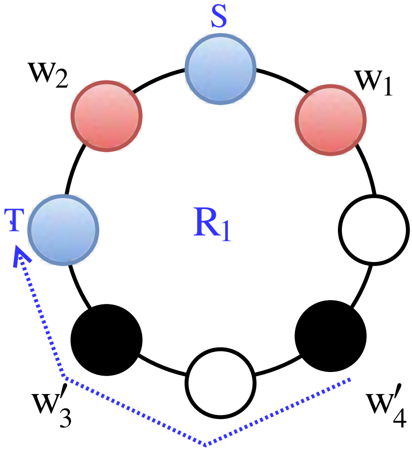

# Summary

Table: Ordered WRP

| | General | $k \in \mathcal{O}(1)$ | Tree | $c_e \in \mathcal{O}(1)$ |
|---|---|---|---|---|
| Feasibility | open | P | P | Ring$\in$ P |
| Optimality | NP-Hard | open | | |

Table: Unordered WRP

| | General | $k \in \mathcal{O}(\frac{\log n}{\log \log n})$ |
|---|---|---|
| Feasibility | P | |
| Optimality | NPH,APX,FPT | P |

# Open Questions

- Other special graph classes, e.g.: bidirected planar graphs
- Feasibility hardness for the ordered variant (we gave the optimality hardness)