A Self-Stabilizing and Local Delaunay Graph Construction

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18. December 2009

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Motivation

Peer-to-Peer Network

- Decentralized
- Recover from faults

⇒ Self-stabilizing

- Nice geometric properties

Motivation

Peer-to-Peer Network

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Related Work

- 1-D graphs: Line, ring, and skip graphs
- 2-D graphs: Delaunay graph in wireless systems

Delaunay Graphs

Assume nodes in general position.

Definition (Delaunay Graph G_D)

$\{u, v\} \in G_D$ iff some circle C contains u, v but no other nodes.

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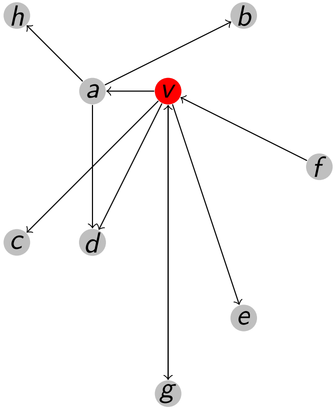
Properties

- Triangulation
- Geometric spanner
- Allows greedy routing

Model (1)

- Only direct neighbors known



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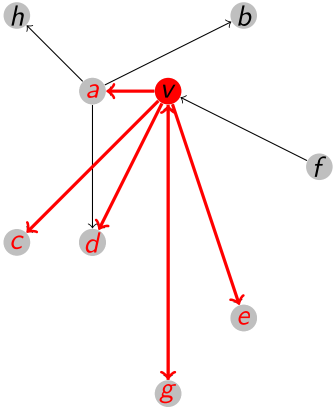
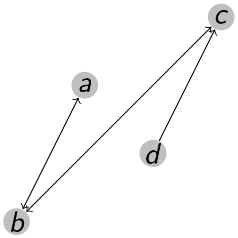


Model (2)

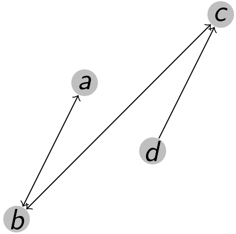
- Updates in rounds



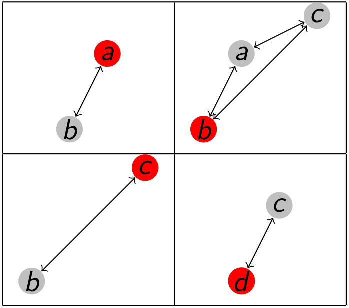
Initial graph

Model (2)

- Updates in rounds



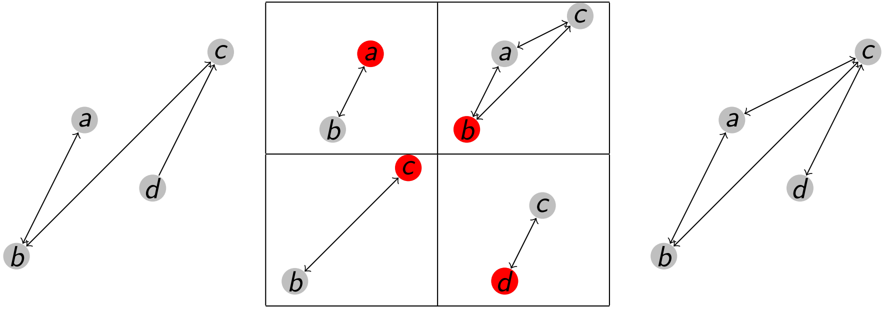
Initial graph



Local updates

Model (2)

- Updates in rounds



Initial graph   Local updates   Update graph

Problem Statement

Situation

- Start with weakly connected graph
- Nodes in general position

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Task

- Converge to Delaunay graph
- Little resources (number of edges)
- Monotonous behaviour

Trivial Strategy

1. If local incorrectness appears, calculate complete graph: $O(n)$
2. Compute Delaunay graph.

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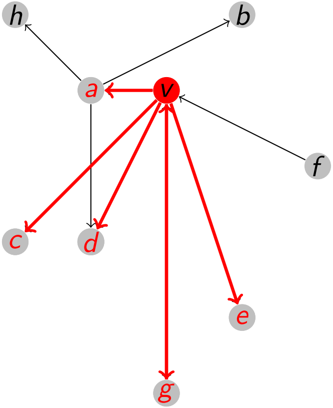
Criticism

- Too many edges
- Not monotonous

Local Update

Compute Delaunay graph of neighbors of v.



Local Update

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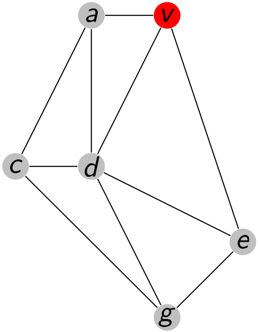
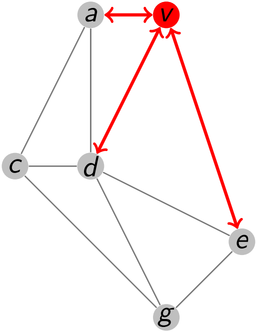


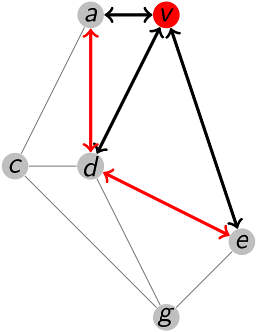
Rule I (1)

Select edges to local neighbors of v.



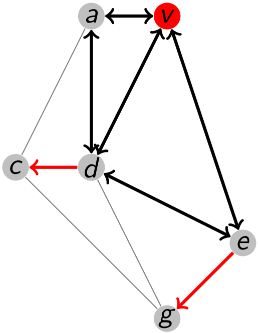
Rule I (2)

Select circular edges around v.



Rule II

Connect local non-neighbors (nearest neighbor strategy).
"Temporary edges"



Main Theorem

Theorem

Using this algorithm, any weakly connected graph G converges in $O(n^3)$ rounds to the Delaunay graph.

If G is a super graph of the Delaunay graph, it converges in $O(n)$ rounds.

Proof Idea

- Delaunay edges don't disappear.
- Temporary edges (Rule II) shorten in each step.
- Define potential $\phi(G)$ which
  - has only $O(n^2)$ values,
  - decreases at least every n steps, and
  - is zero if and only if G is a super graph of the Delaunay graph.
- Superfluous edges disappear in $n - 1$ rounds.

$\Rightarrow$ Convergence in $O(n^3)$ rounds

Example Proof

Lemma

The only 'stable' graph is the Delaunay graph.

Proof.

- 'Stable' means 'locally triangulated'.
- 'Locally triangulated' implies planar.
- Planar and 'stable' implies triangulated.
- From any triangulation, edge flips lead to the Delaunay graph.

$\Rightarrow$ 'Stable' implies Delaunay graph.

□

Experiments

Setting

- Random graphs
- Up to 500 nodes
- Various topologies:
  - Random tree
  - Maximum spanning tree
  - Clique
  - Slightly disturbed circle

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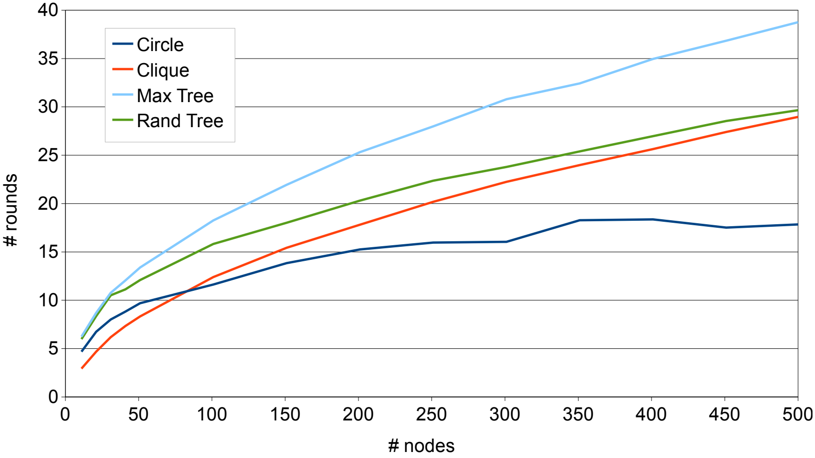
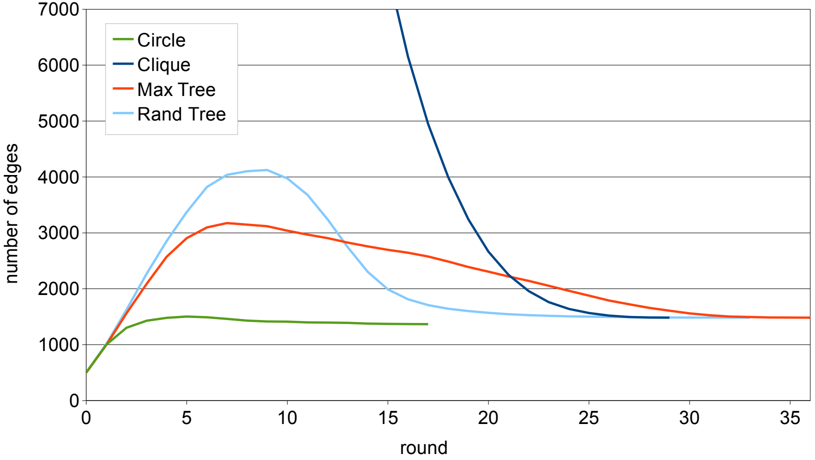
Measurements

- Number of rounds
- Number of edges

Statistics: Runtime



Statistics: Number of edges



Results

- Run time much better than analysis
- Number of edges can rise in first rounds, but not too much
- Results stable for randomly distributed nodes
- High variance for CIRCLE topology

Conclusion

- Self-stabilizing Delaunay graph construction
- Worst case time bound: $O(n^3)$ rounds
- Experiments on random graphs:
  - Less than n rounds
  - Moderate edge growth

Open Problems

- Better bound on number of rounds
- Bounds for edge growth
- Construct worst case examples
- Alternative strategies (Rule II)
- Resolve round scheduling

Thank you!